Observatoire de la CÔTE d'AZUR

Stellar radii from interferometry

D. Mourard - OCA France

Outline

1. Basic considerations about radius, diameter, Teff
2. Rapid introduction to the interferometric measurements
3. Could we measure the angular diameter of stars, and the LD?
4. What is the current situation?
5. What are the perspectives? CHARA/SPICA project

Basic considerations

R = Radius (m, $R_\odot$)

θ_{LD} = Angular diameter (millisecond of arc == mas)
it should be understood as the LD diameter, not the equivalent Uniform Disk diameter (θ_{UD}) which is wavelength dependent.

$$R = \frac{\theta_{LD}}{9.305\ \pi_p}$$ **with R in $R_\odot$, θ_{LD} in mas and π_p the parallax in second of arc**

<u>Effective temperature</u>

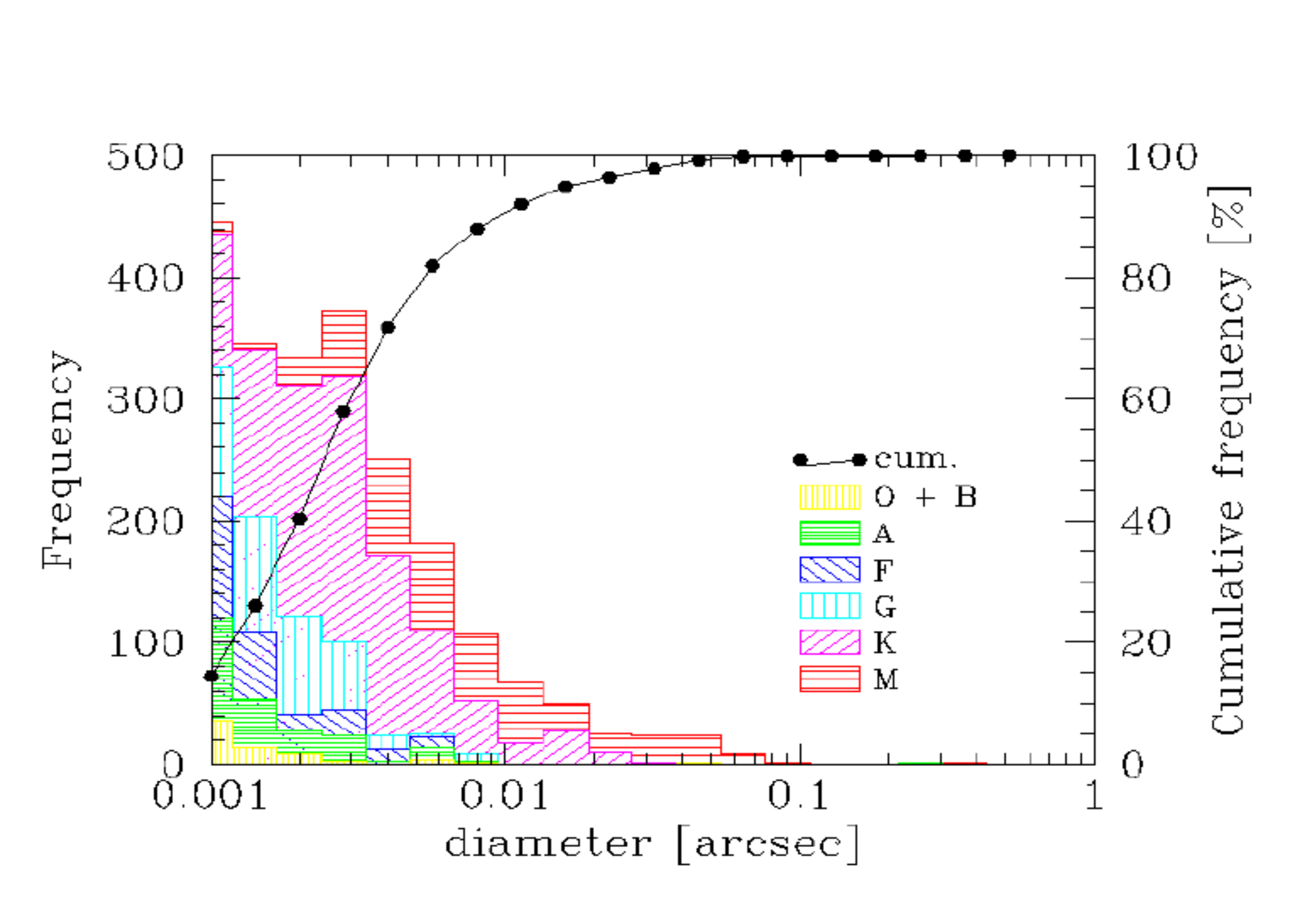
Luminosity & Radius

$$\sigma {T_e}^4 = \frac{L}{4\pi R^2}$$

Bolometric fluc and angular diameter

$$\sigma T_e^4 = \frac{4 f_{bol}}{\theta^2}$$, with θ in radian

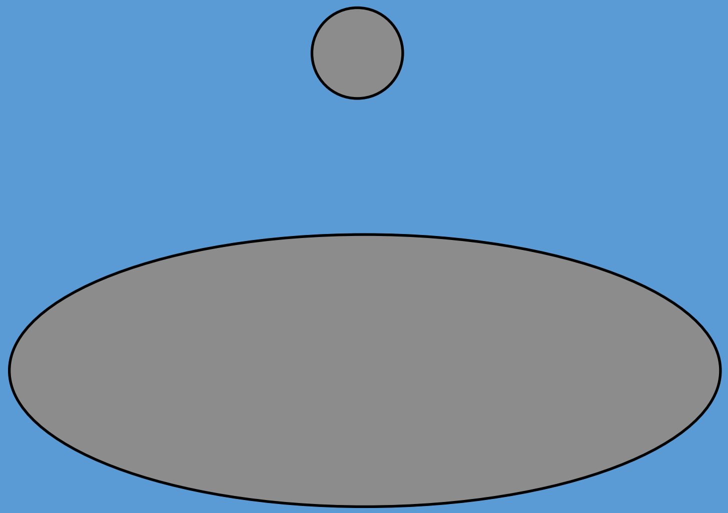
But apparent diameters are small…



1309 sources
50 % < 2.5 mas
20 % > 5 mas
7% > 10 mas -> UT

How to measure a so small angular diameter?

Star at infinity
Angular diameter θ



Screen of radius r

Angular diameter θ ➔ solid angle $\Omega=\pi(\theta/2)^2$

Radius r ➔ Surface $S=\pi r^2$

Etendue of the beam

$$\varepsilon = S\Omega = \pi^2 r^2 (\theta/2)^2$$

Definition of coherence (Goodman)

$$\varepsilon < \lambda^2$$

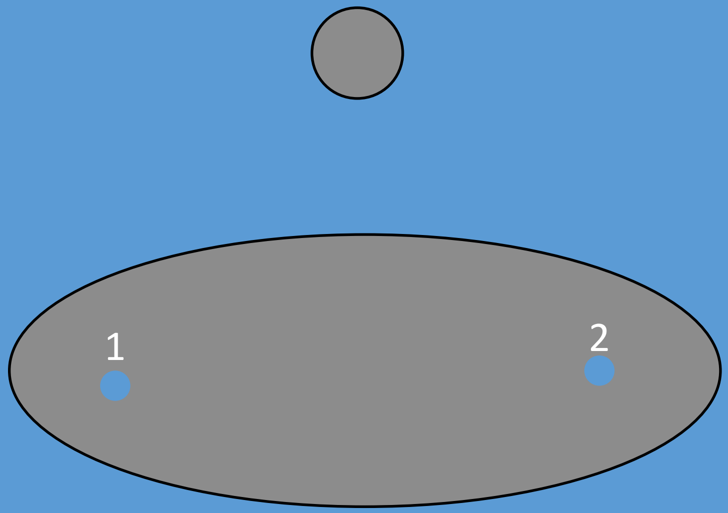
➔

$$r_c = \frac{\lambda}{\pi\left(\frac{\theta}{2}\right)}$$

N.A.: $\theta=10$mas, $\lambda=1\mu$m ➔ $r_c= 13$m

Coherence & Van-Cittert Zernike Theorem

Star at infinity
Angular diameter θ



Screen of radius r

The coherence of the electromagnetic wave could be determined by the computation of the complex degree of mutual coherence between points 1 & 2 of the collecting area:

$$\Gamma_{12} = \frac{\left|\psi_1 \psi_2^*\right|}{\sqrt{\left|\psi_1\right|^2 \left|\psi_2\right|^2}}$$

V-CZ ➔

$$\Gamma_{12} = \frac{\left|\tilde{O}(B/\lambda)\right|}{\left|\tilde{O}(0)\right|}$$

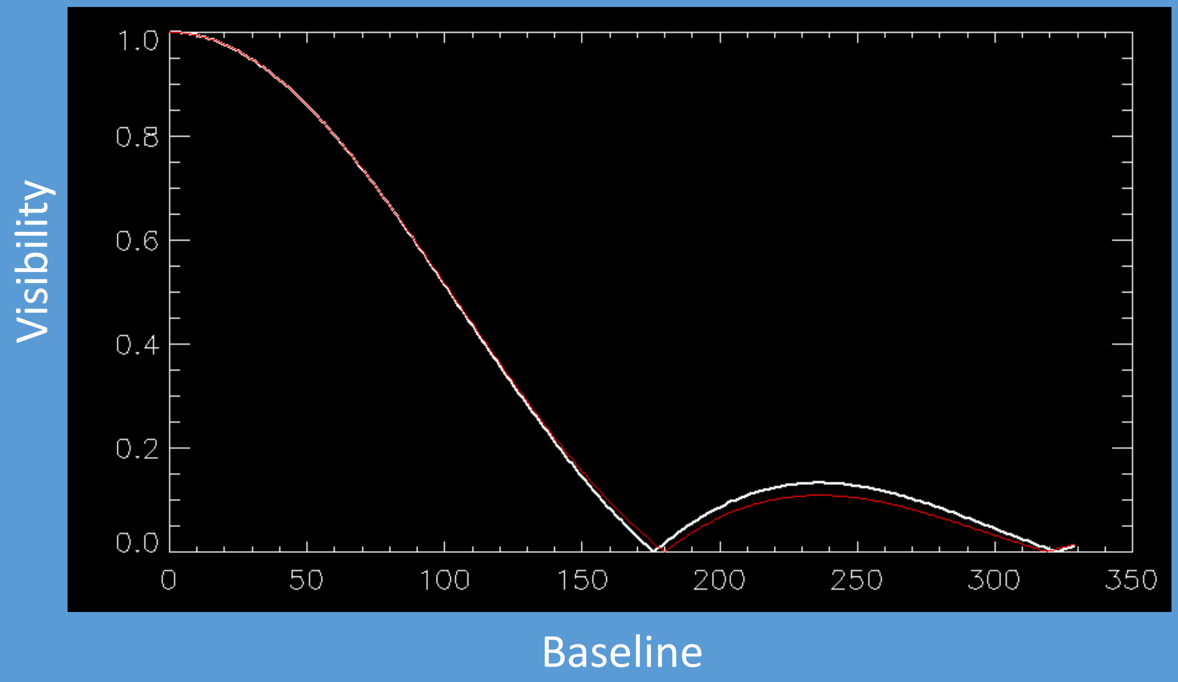
Uniform Disk:

$$\Gamma_{12} = \left|\frac{2J_1(\pi\theta B/\lambda)}{\pi\theta B/\lambda}\right|$$

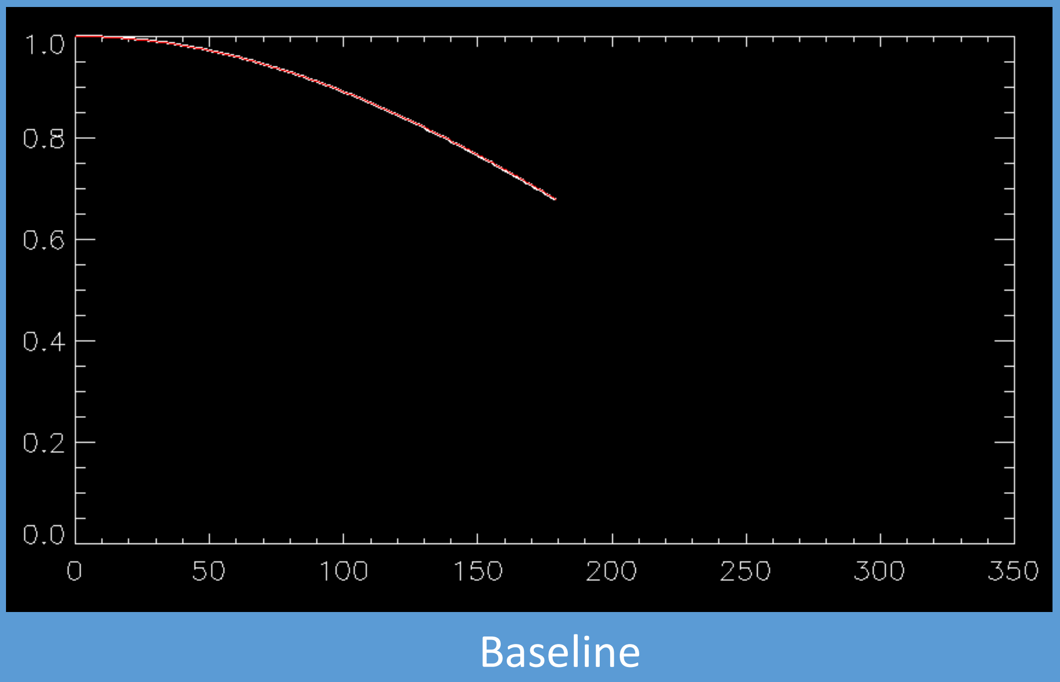
Note: The definition of r_c (Goodman) corresponds at B where $\Gamma_{12}=0.5$ i.e.
$\pi\theta B/\lambda=2$ ➔ $r_c=B=2\lambda/\pi\theta$ ➔ $\varepsilon=\lambda^2$.

Visibility function of a star θ_{UD}=1mas

CHARA, 330m, λ=700nm + θ_{LD}(0.5)



VLTI, 180m, λ=1600nm + θ_{LD}(0.5)



ΔV=1% ➔ $\Delta\theta/\theta$=1%
Sensible to LD

ΔV=1% ➔ $\Delta\theta/\theta$=2.5%
Insensible to LD

Limb darkening & interferometry – 1st basic considerations

Mon. Not. R. astr. Soc. (1974) **167**, 475–483.

THE EFFECTS OF LIMB DARKENING ON MEASUREMENTS OF ANGULAR SIZE WITH AN INTENSITY INTERFEROMETER

R. Hanbury Brown, J. Davis, R. J. W. Lake and R. J. Thompson

2. A SIMPLE REPRESENTATION OF LIMB DARKENING

In the conventional linear representation of limb darkening the distribution of brightness across the star's disc is given by,

$$I_\lambda(\mu) = I_\lambda(1)[1 - u_\lambda(1-\mu)] \quad (3)$$

where $I_\lambda(\mu)$ is the brightness of a point on the disc at a wavelength λ, μ is the cosine of the angle between the normal to the surface at that point and the line of sight from the star to the observer, and u_λ is the limb-darkening coefficient. By taking the Hankel transform of the apparent angular distribution of intensity across the source it can be shown that,

$$\Gamma_\lambda^2(d) = (\alpha/2 + \beta/3)^{-2}[\alpha J_1(x)/x + \beta(\pi/2)^{1/2} J_{3/2}(x)/x^{3/2}]^2 \quad (4)$$

where $\alpha = 1 - u_\lambda$, $\beta = u_\lambda$, $x = \pi\theta_{LD}\, d/\lambda_0$, θ_{LD} is the true angular diameter of the limb-darkened star, and it is assumed that $\Delta_\lambda = 1$.

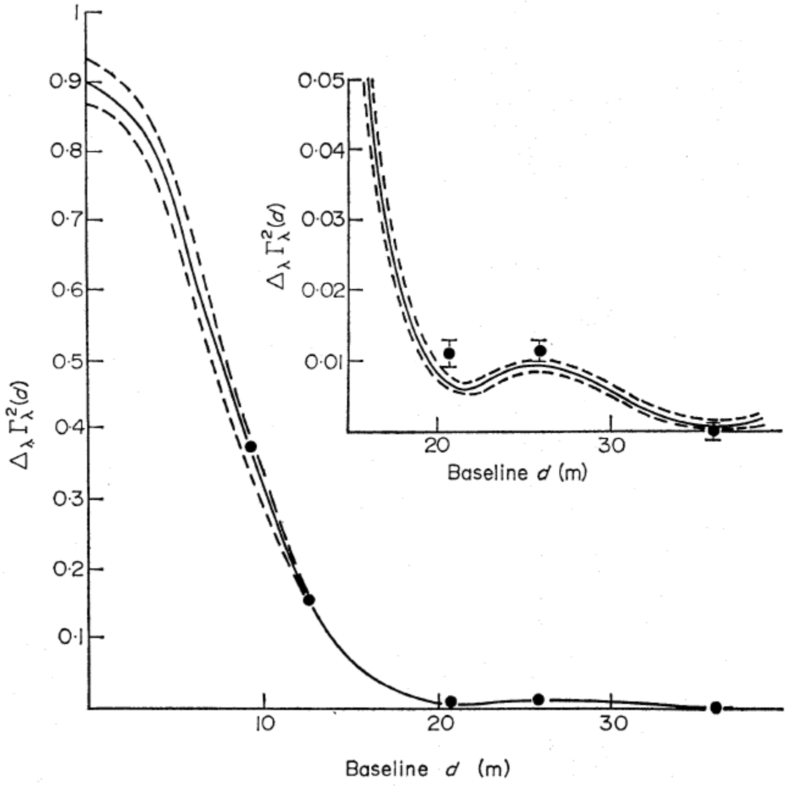


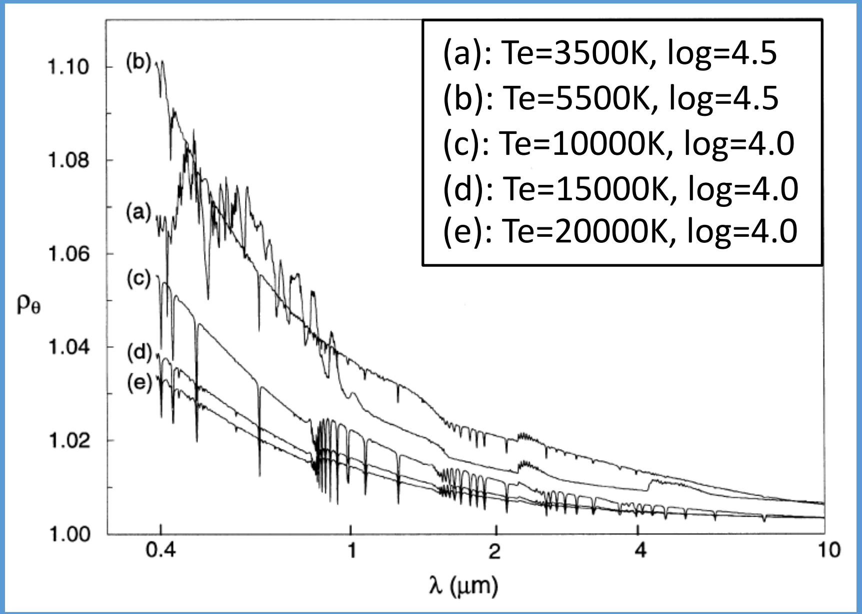
FIG. 3. *The variation of correlation $\Delta_\lambda\Gamma_\lambda$ (d) with baseline d for Sirius. The points show the observed values; the full line is a theoretical curve, based on a model stellar atmosphere (T_e = 10 000K, log g = 4), with zero-baseline correlation and angular size adjusted to give the best fit to the observations. The broken lines represent the rms uncertainty in the theoretical curves.*

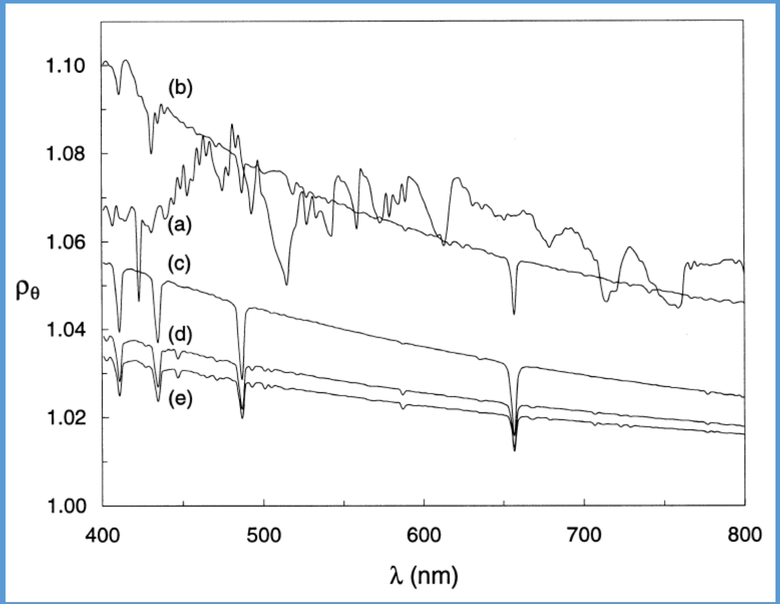
Limb darkening & interferometry – "classical considerations"

Limb-darkening corrections for interferometric uniform disc stellar angular diameters

J. Davis,★ W. J. Tango and A. J. Booth† Mon. Not. R. Astron. Soc. **318**, 387–392 (2000)

$$\rho_\theta = \theta_{\rm LD}/\theta_{\rm UD}$$





Measure of θ_{UD} + estimation of **u** the linear limb darkening coefficient (Te, logg) ➔ $\rho_\theta = \sqrt{\frac{1-\frac{u}{3}}{1-\frac{7u}{15}}}$ ➔ θ_{LD}

Different definitions of Limb Darkening

From Kervella et al., A&A 597 (2017)

- uniform disk:

$$I(\mu)/I(1) = 1; \quad (2)$$

- linear:

$$I(\mu)/I(1) = 1 - u\,(1-\mu); \quad (3)$$

- power law (Hestroffer 1997):

$$I(\mu)/I(1) = \mu^{\alpha}; \quad (4)$$

- quadratic:

$$I(\mu)/I(1) = 1 - a\,(1-\mu) - b\,(1-\mu)^2; \quad (5)$$

- square root:

$$I(\mu)/I(1) = 1 - c\,(1-\mu) - d\,(1-\sqrt{\mu}); \quad (6)$$

- four-parameter:

$$I(\mu)/I(1) = 1 - \sum_{k=1}^{4} a_k \left(1 - \mu^{k/2}\right). \quad (7)$$

In addition, we consider the following polynomial model with six parameters:

- polynomial:

$$I(\mu)/I(1) = \frac{\sum_{k=0}^{5} s_k\,\mu^k}{\sum_{k=0}^{5} s_k}. \quad (8)$$

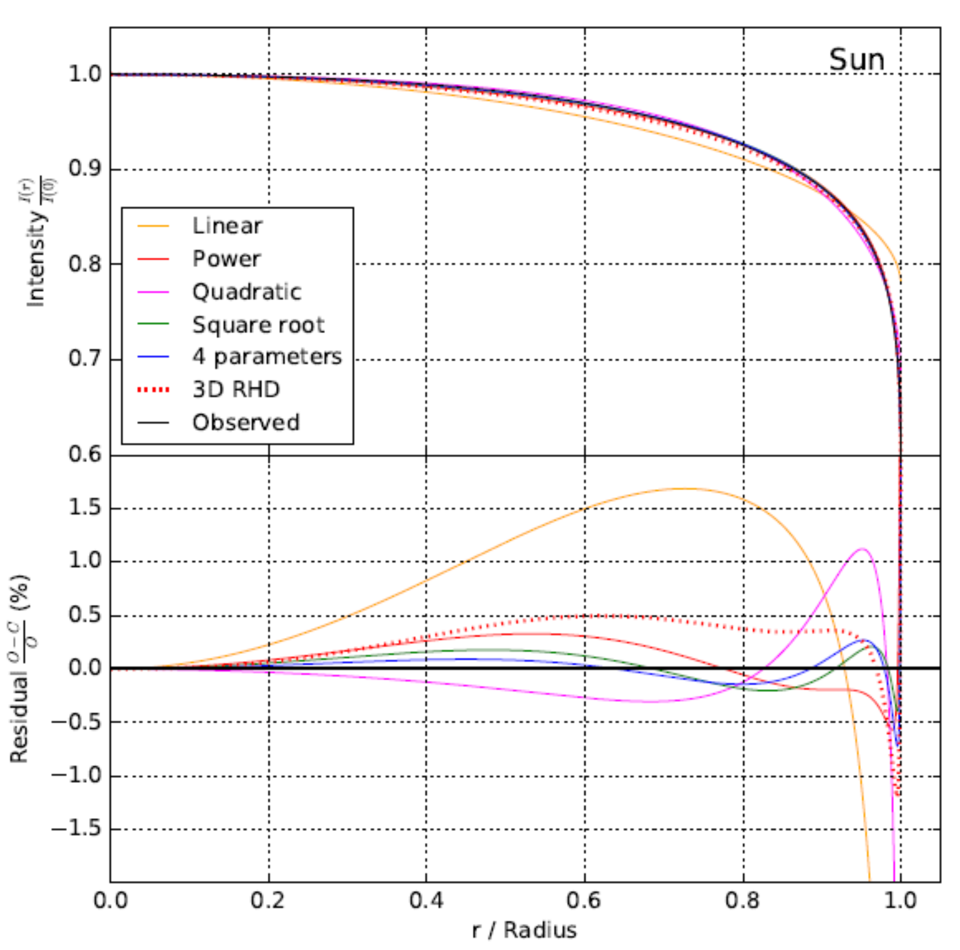


Fig. 2. Comparison of different parametric limb darkening models of the Sun with the observed limb darkening profile measured by Pierce et al. (1977) in the H band. The residuals in percentage of the observed intensity profile are shown in the *lower panel*.

Actual measurements with VLTI/PIONIER

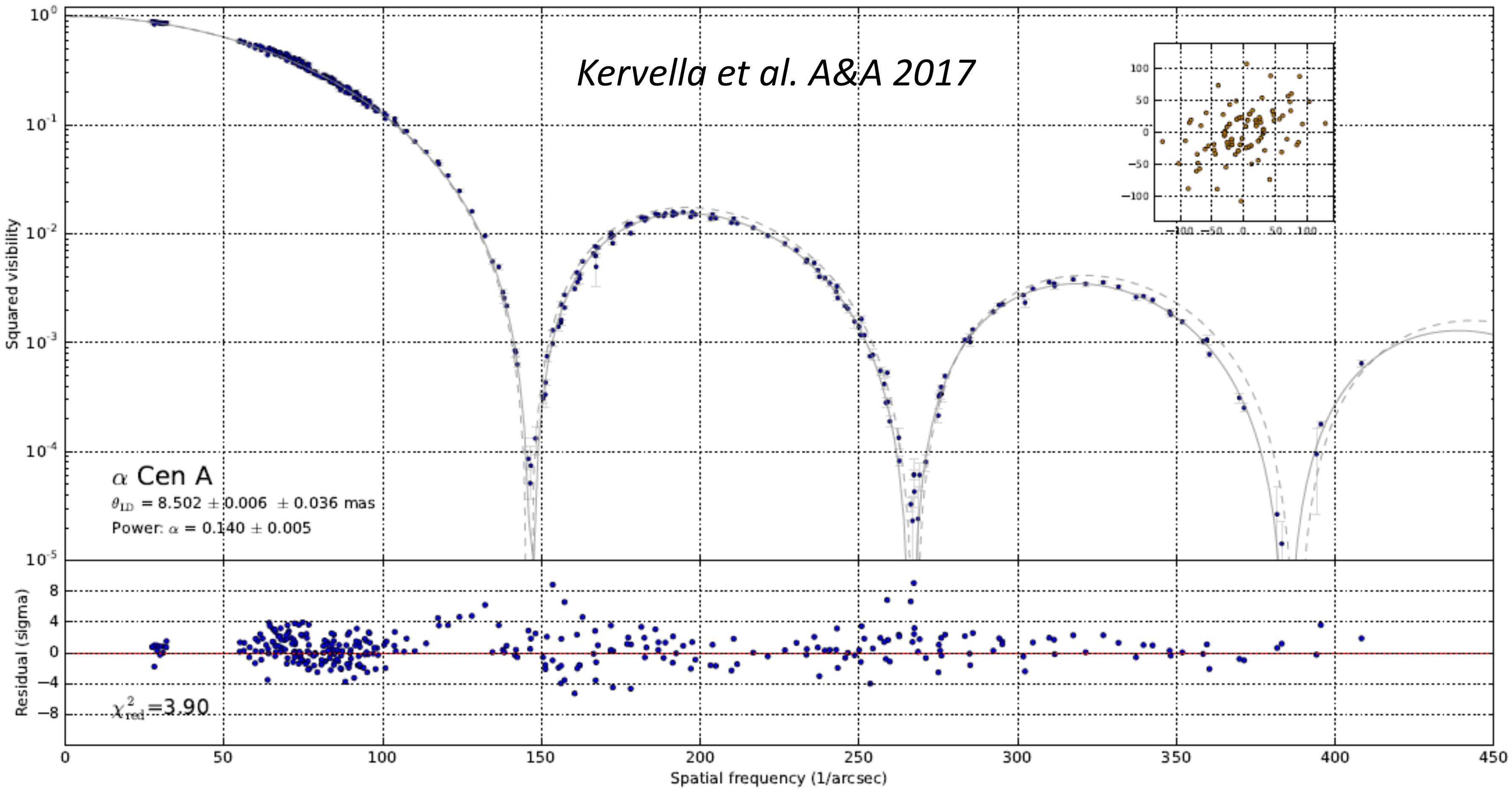


Fig. 3. Adjustment of a power law limb darkened disk model to the PIONIER squared visibilities of α Cen A (solid gray curve). The dashed gray curve represents the best-fit uniform disk model. The *bottom panels* show the residuals of the fit in number of times the statistical error bar. The coverage of the (u, v) plane is shown in the *upper right corner.*

And α Cen B

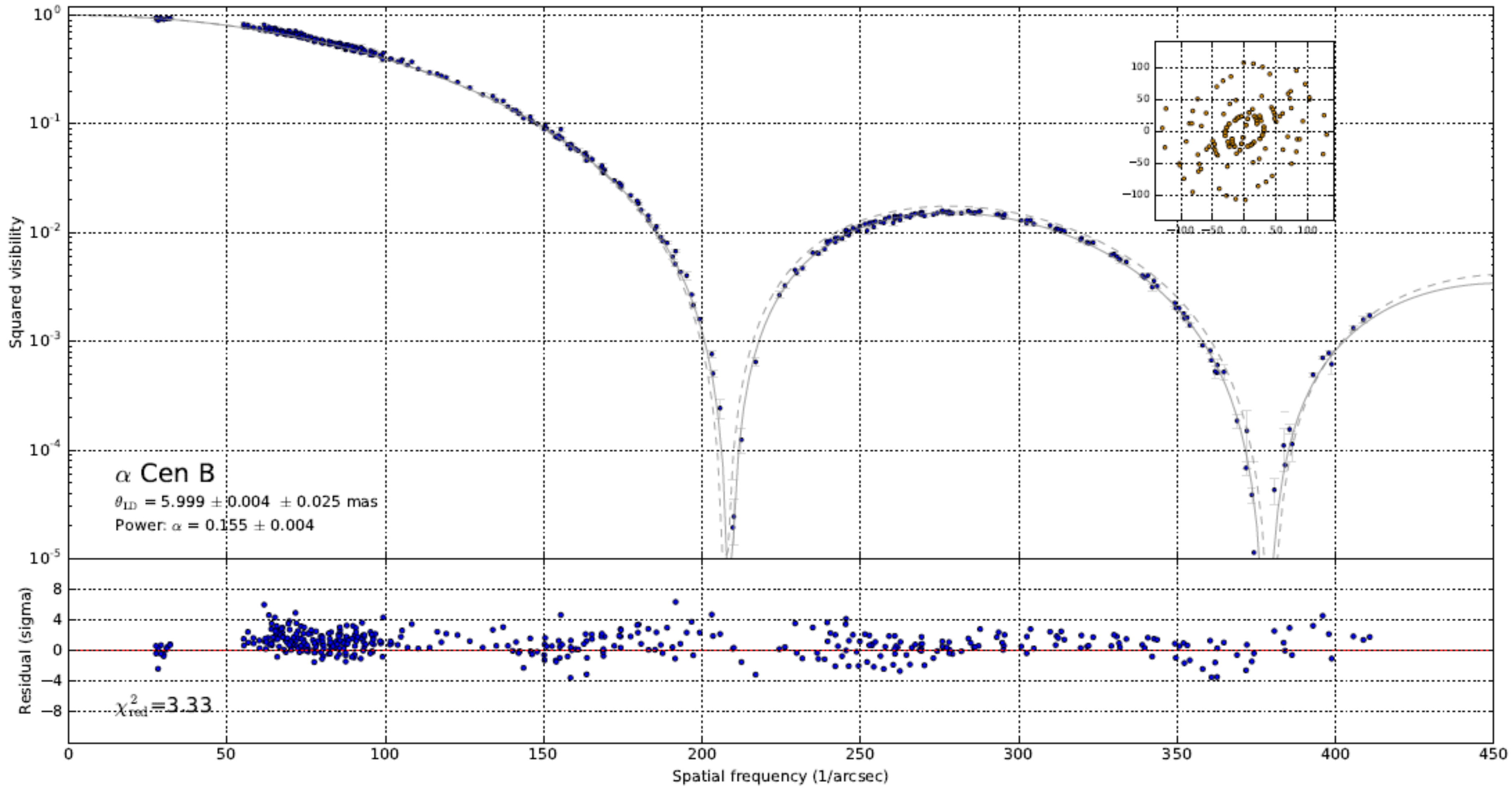
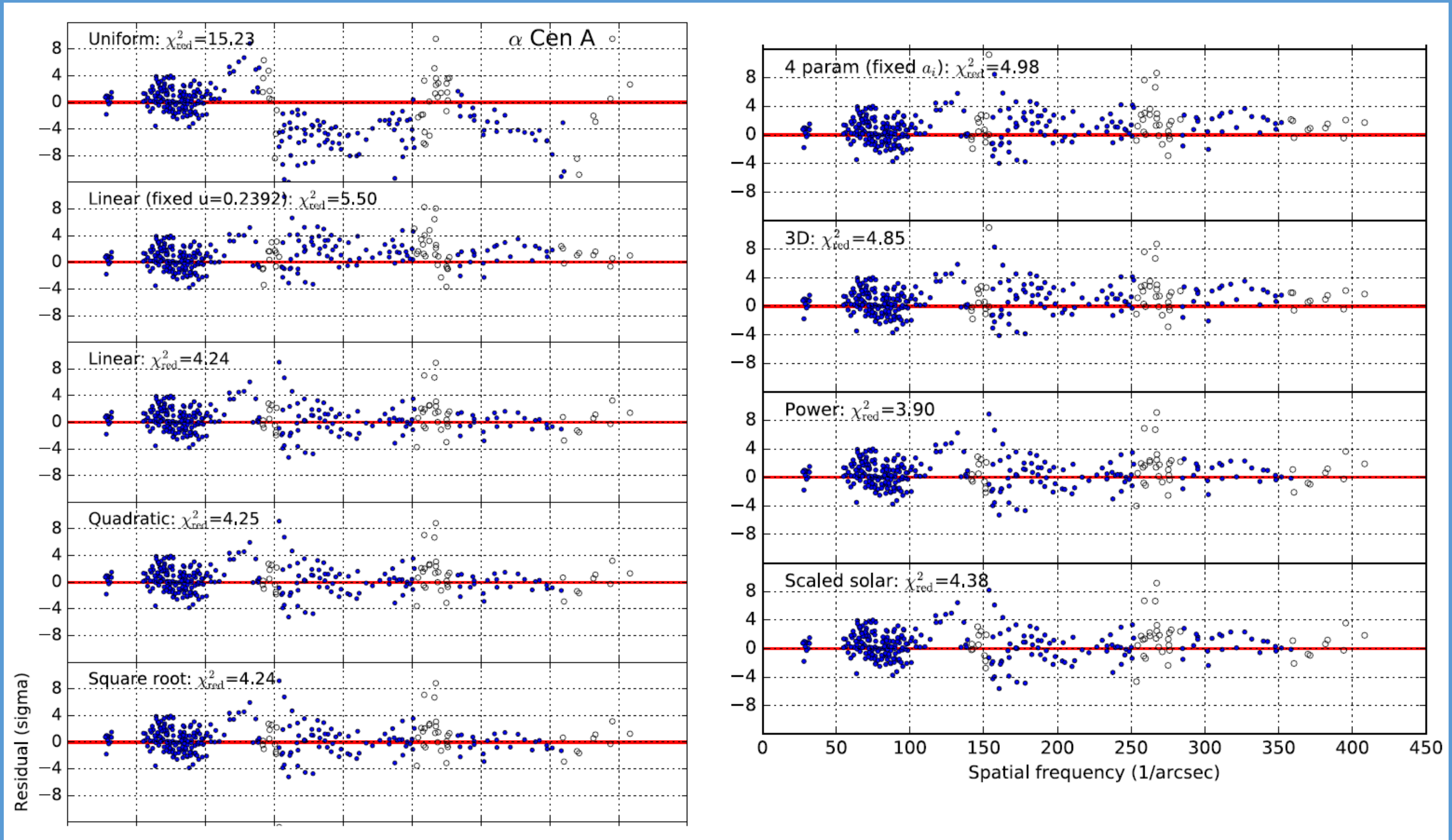
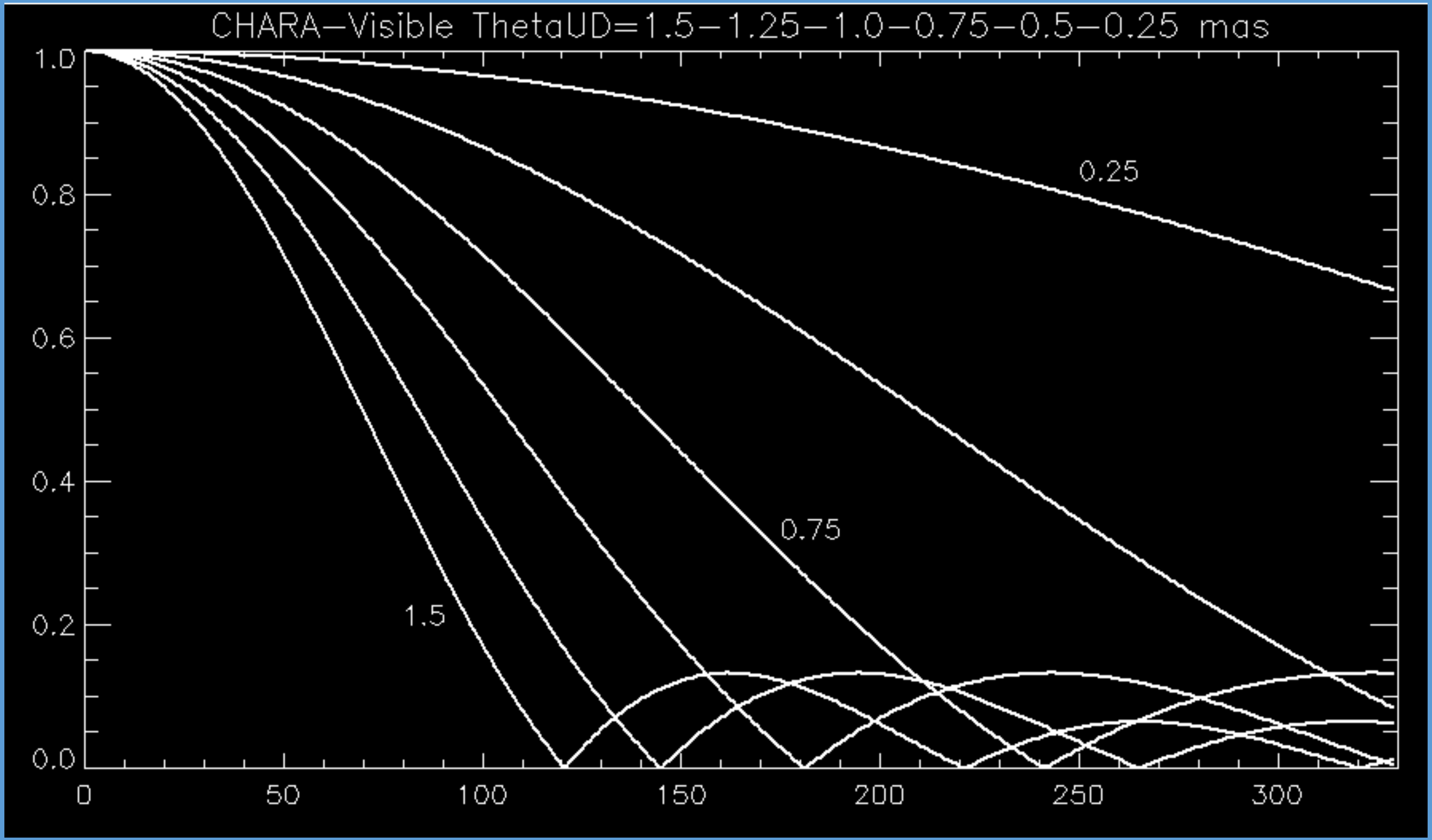


Fig. 4. Power law limb darkened disk model fit and residuals for α Cen B (same caption as Fig. 3).

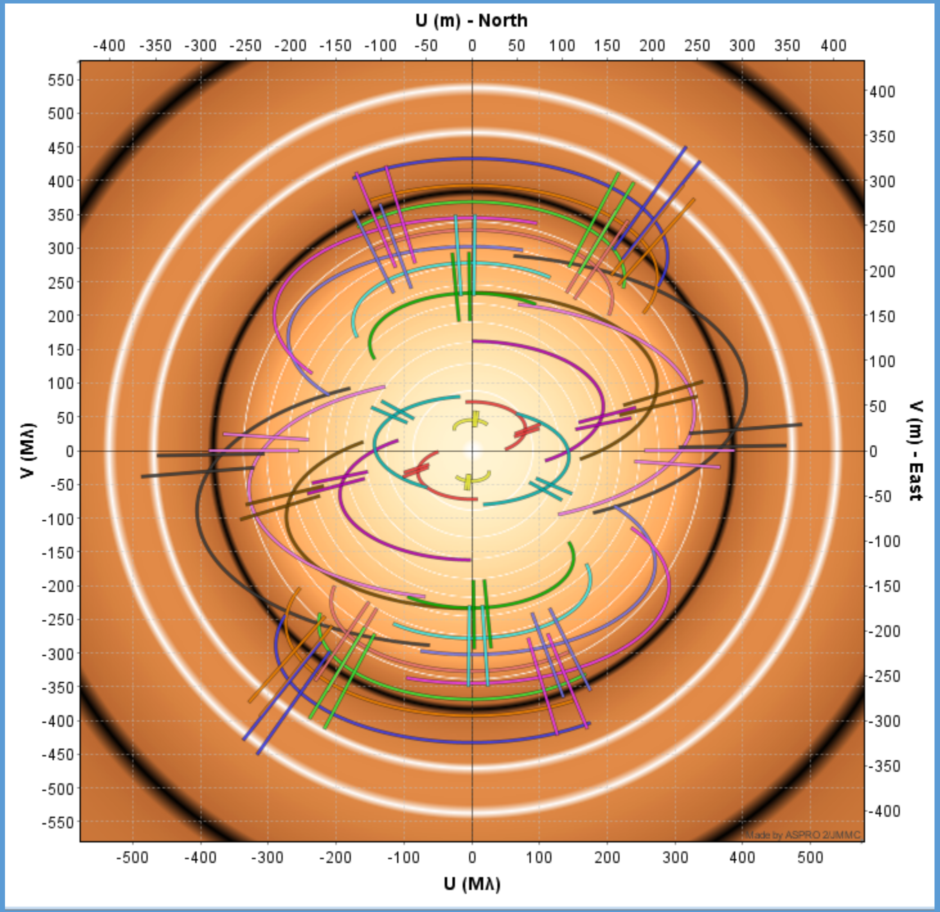
Comparison of the residuals on α Cen A

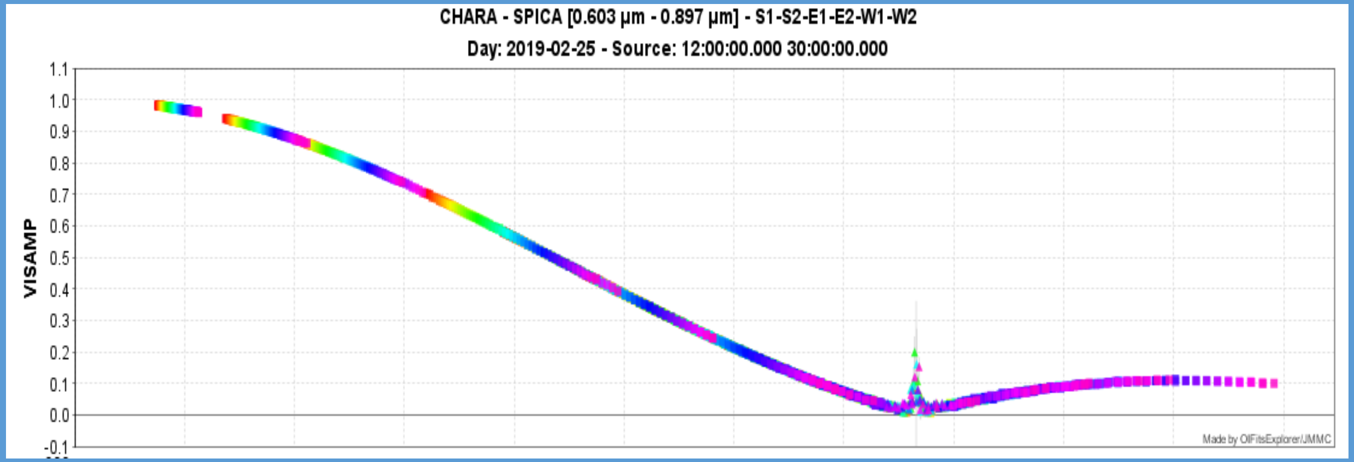


CHARA array, visible wavelengths - V^2 functions



θ_{LD}=0.7mas, u=0.5 *(ASPRO2 & LITpro tools JMMC)*

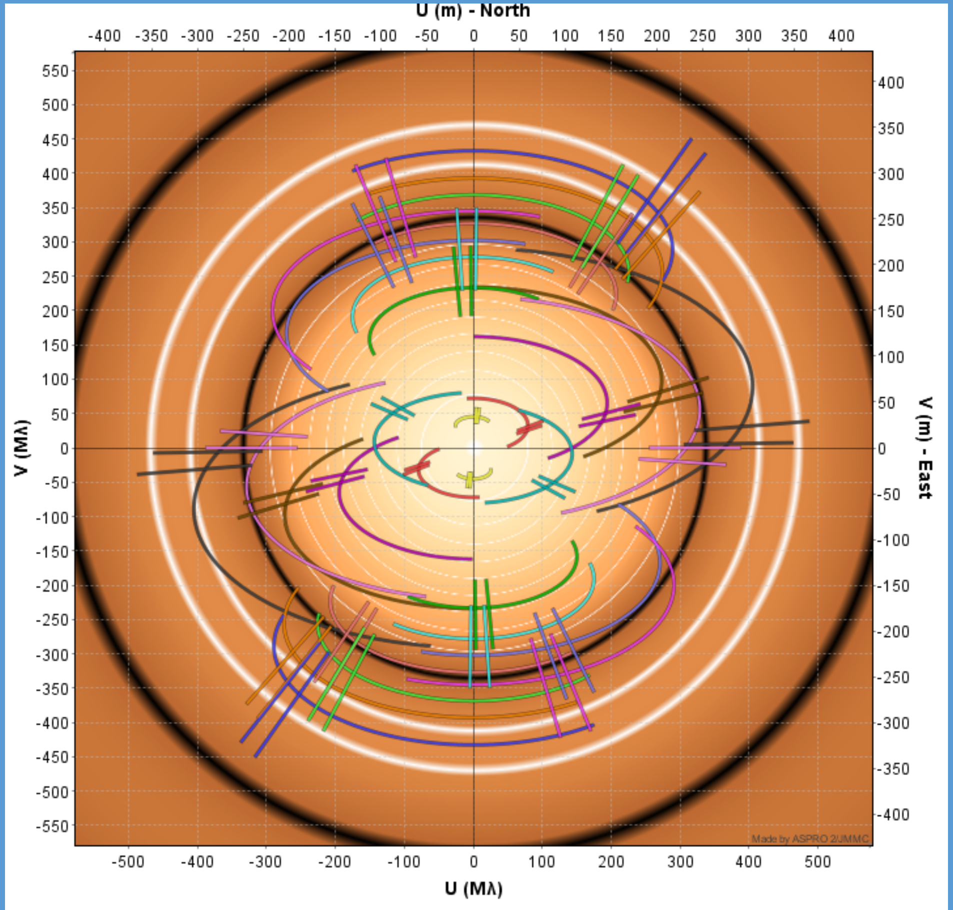


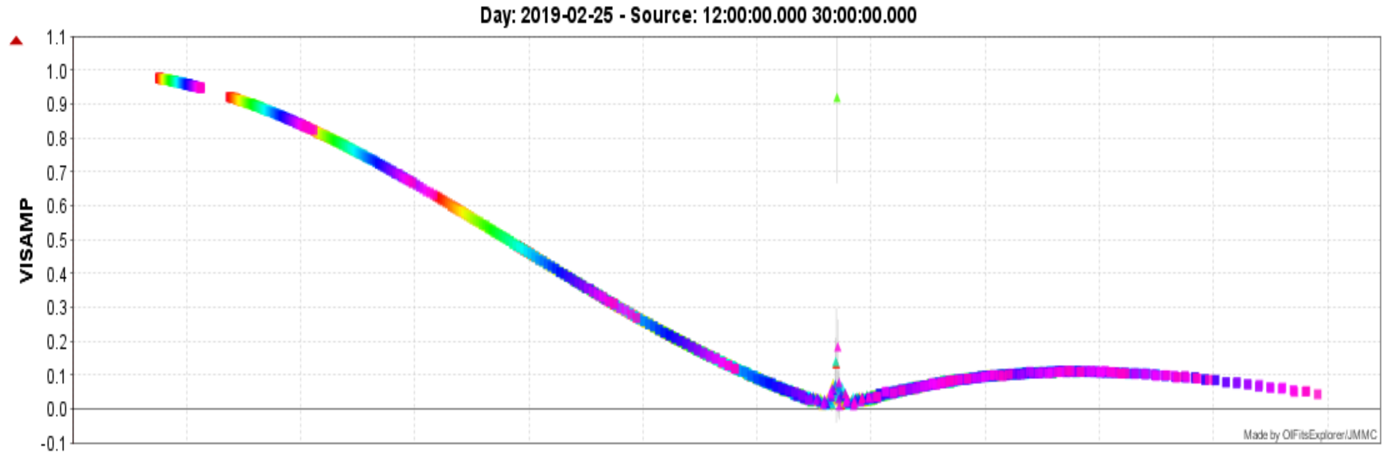


θ_{UD}= 0.6664 +-0.0004

θ_{LD}= 0.6962 +-0.0003, u=0.522+-0.004

θ_{LD}=0.8mas, u=0.5 *(ASPRO2 & LITpro tools JMMC)*





θ_{UD}= 0.7611 +-0.0002

θ_{LD}= 0.8004 +-0.0002, u=0.504+-0.002

First conclusions on LD measurements

For CHARA and visible: θ_{LD}=0.8mas is ok for a direct measure of LD

For VLTI/PIONIER, the same simulation leads to θ_{LD}=4mas

So it's very clear that only very few stars could be directly measured in terms of LD diameter with the VLTI

Estimation of θ_{LD} for dwarfs

magV	F	G	K
4	0,892	1,332	1,935
5	0,563	0,840	1,221
6	0,355	0,530	0,770

➔on CHARA, magV=4.2 for FV, magV=5.1 for GV, and magV=5.9 for KV

➔~ 100 F stars, 200 G stars, and 800 K stars.

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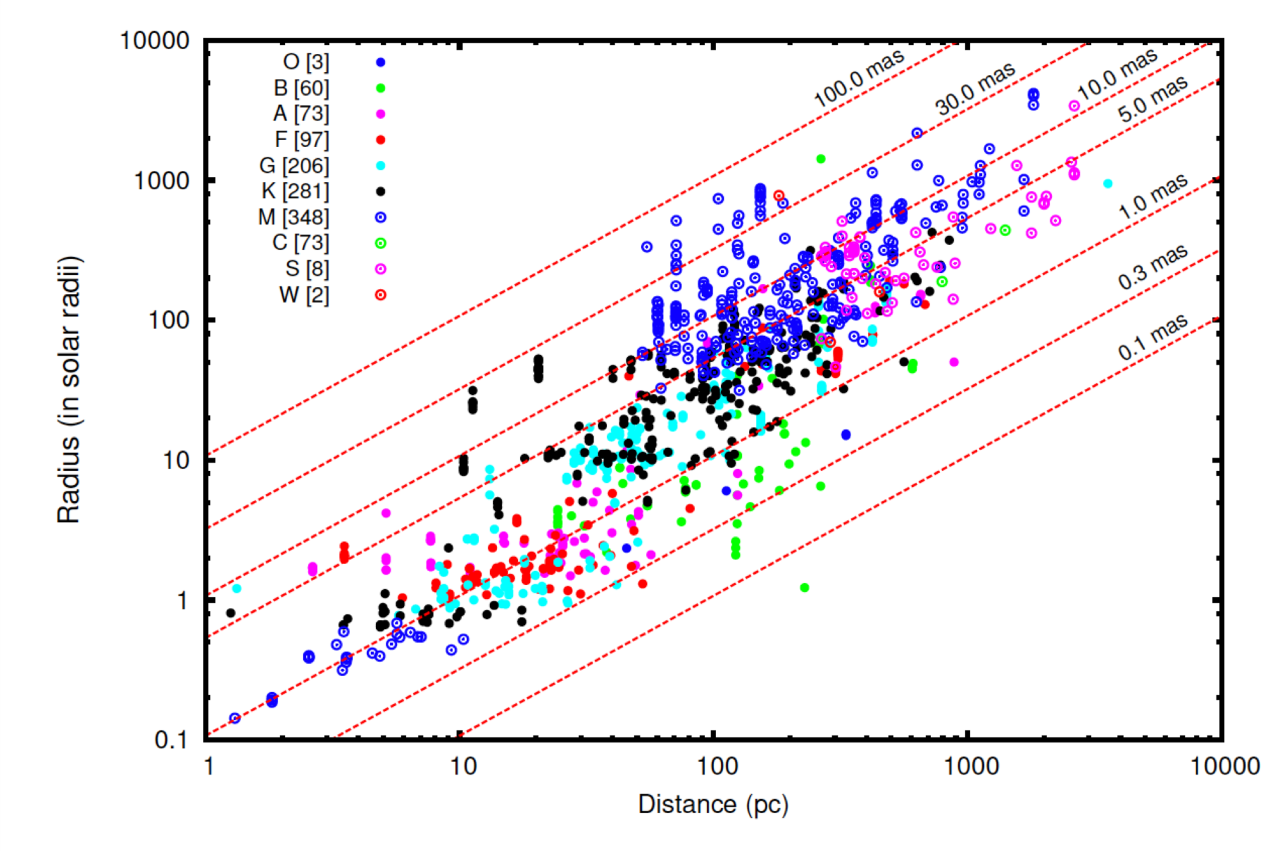
The JMDC catalog (➔Vizier, Duvert+2016)

- ~1500 stellar angular diameter measurements from different techniques (lunar occultation, intensity interferometry and optical interferometry).
- 11% (resp. 22%) of stars have their angular diameter measured with a precision better than 1% (resp. 2%). It corresponds to 159 and 323 measurements, respectively.

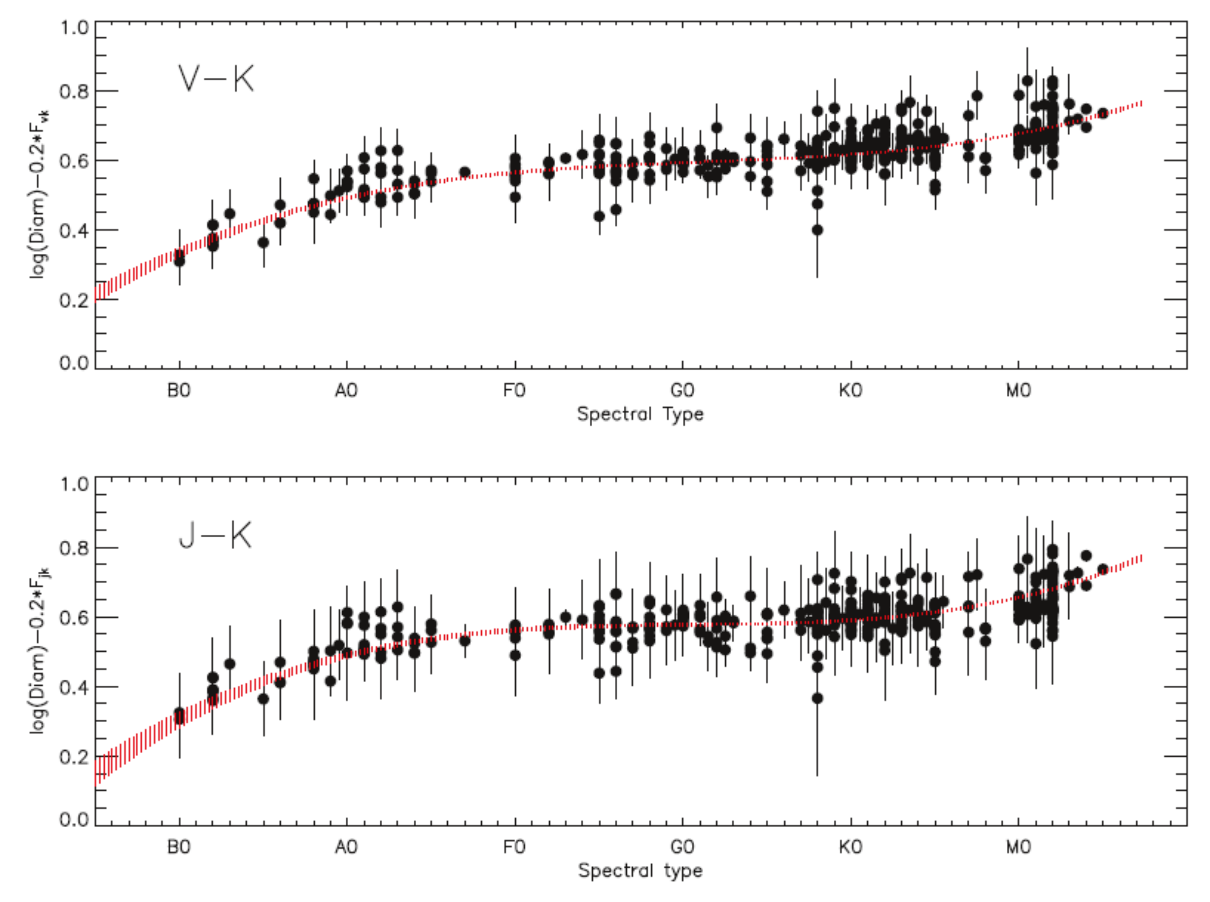


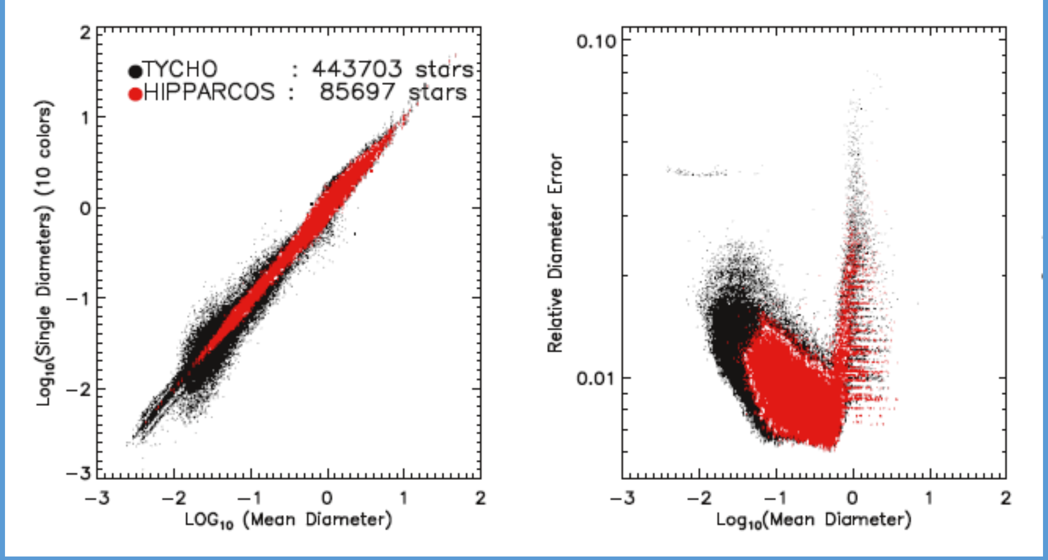
II/346 JMMC Stellar Diameters Catalogue - JSDC. Version 2 (Bourges+, 2017) figures

2014ASPC..485..223B ReadMe+ftp Similar Catalogs

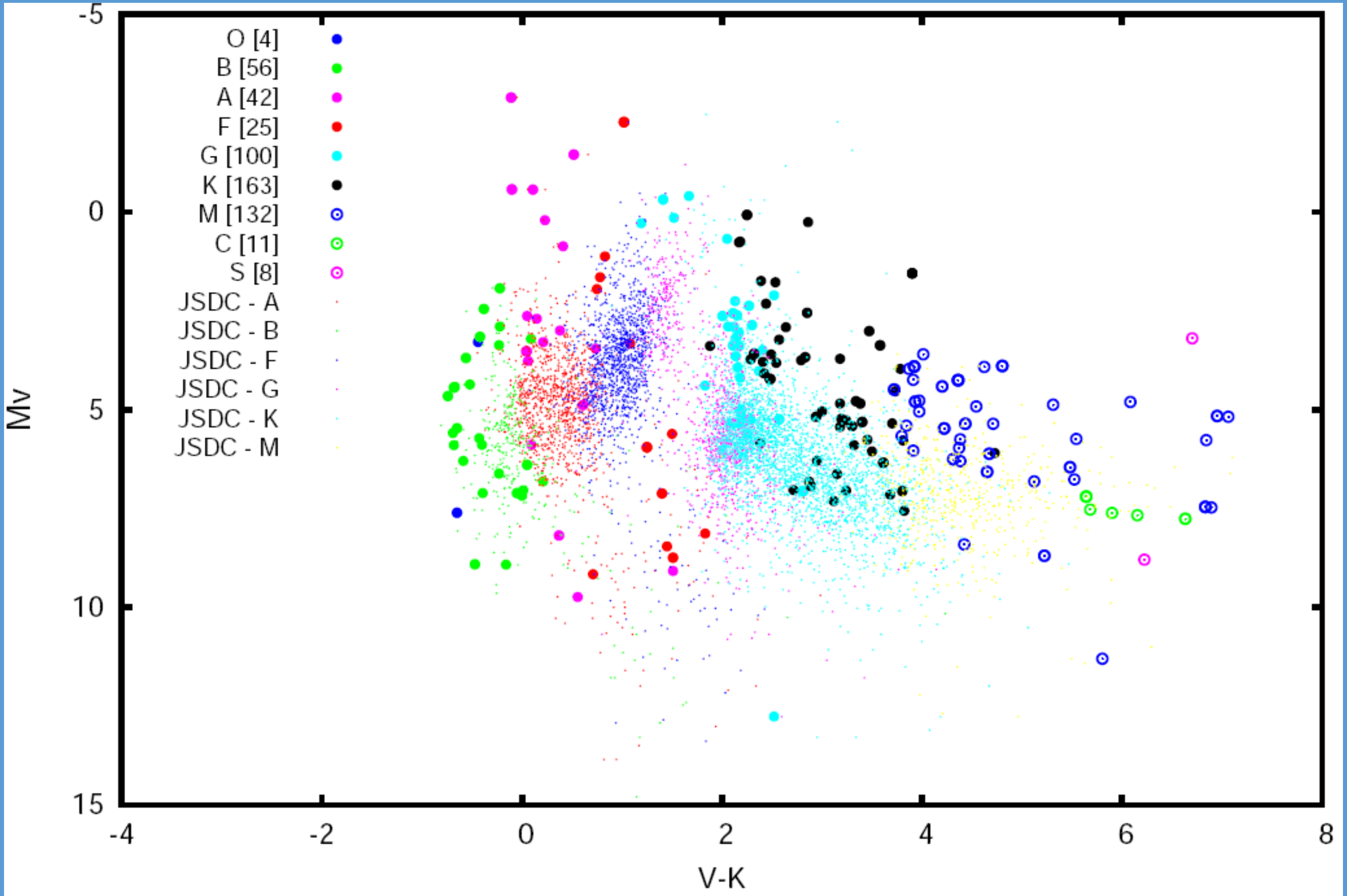
Post annotation

1.II/346/jsdc_v2 Stellar diameters catalogue, version 2 *(465877 rows)*



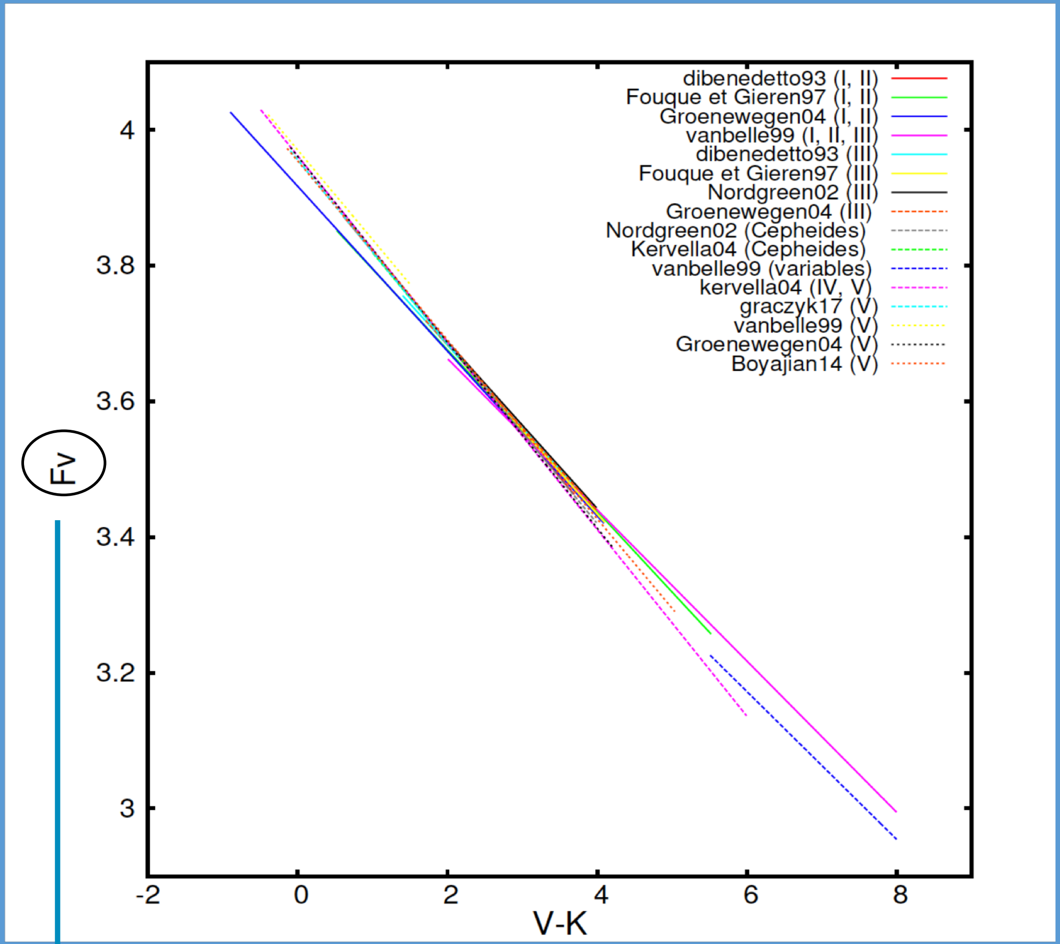


(Mv,V-K) diagram for JMDC+JSDC



But issues are remaining…

Indeed, the JSDC (Chelli+16) provides the angular diameter of 453000 stars with a median statistical uncertainties of 1.1%
But, if we consider the 23 surface-brightness color relations (SBCR) available in the literature, we have inconsistencies



Linked to mV and the angular diameter

1. $S_V = V - 5\log\theta_{LD} = \sum a_k (V-K)^k$
2. $F_V = 4.2207 - 0.1 S_V = \alpha + \beta(V-K)$
3. $\log\theta_{LD} = d_1 + c_1 1(V-K) - 0.2V$
4. $\theta_{LD}(V=0) = 10^{A+B(V-K)}$
5. $\Phi_V = \frac{\theta}{9.305*10^{\frac{-V}{5}}} = \sum z_k (V-K)^k$

If we apply the 23 SBCR to an hypothetic star of mV=6; we obtain a dispersion of :

- 2% if V-K=3
- 9% if V-K=0 (early-type stars)
- 9% if V-K=5 (late-type stars)

Conclusion: We are probably far from being able to estimate the angular diameter of stars with a 1% precision and accuracy.

Diagnosis and solution

- The 23 SBCR are based on various types of data and the methods used are also different.
- The subsets of data used are also very heterogeneous. Indeed, the 23 SBCR are based on samples of stars of 18 to 239 stars.
- There is also the problem of the V and K photometry. We need homogeneous data.
- And physically, as soon as the star is not a black body, we can have potentially a deviation from the SBCR. In other words stellar activity (spots, convection, winds & environment, rotation, and multiplicity) should be also taken into account.

With CHARA/SPICA:

- We can derive the angular diameter of 800 stars with a 1% precision (or better).
  - This would double the number of stars for which we have an angular diameter.
  - This would increase by a factor 5 the number of stars for which we have a 1% precision
  - It would provide a unprecedented sample of stars with homogeneous angular diameters
- We can do images and/or characterize the stellar activity of ~200 stars

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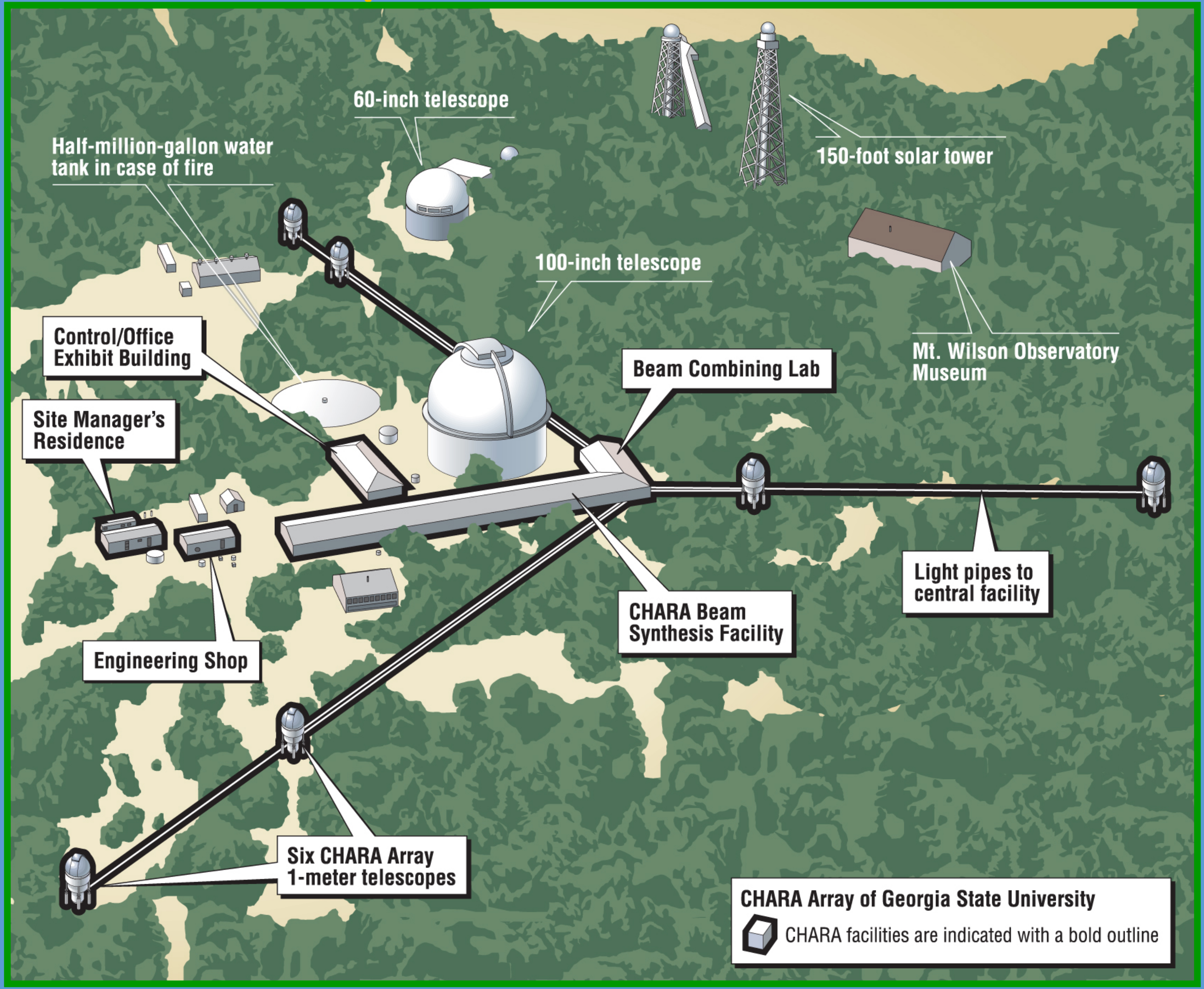
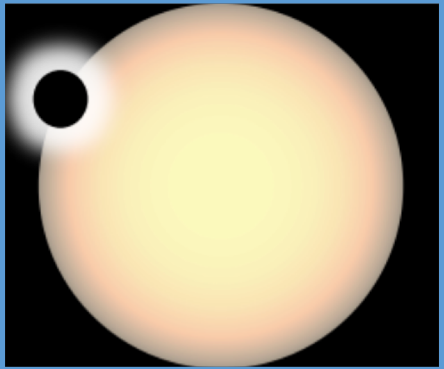
The CHARA Array - Mount Wilson Observatory

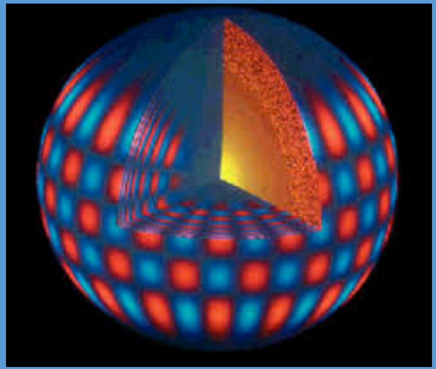


Why is it crucial to derive the angular diameter of stars with a 1% precision and accuracy ?

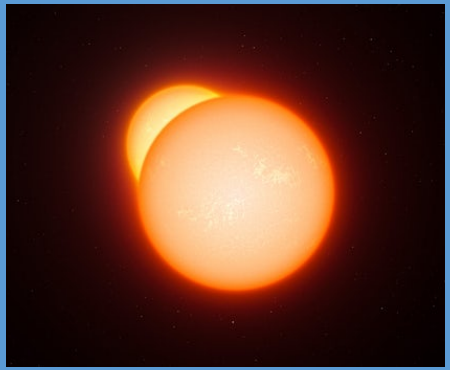
Three astrophysical objectives of CHARA/SPICA:



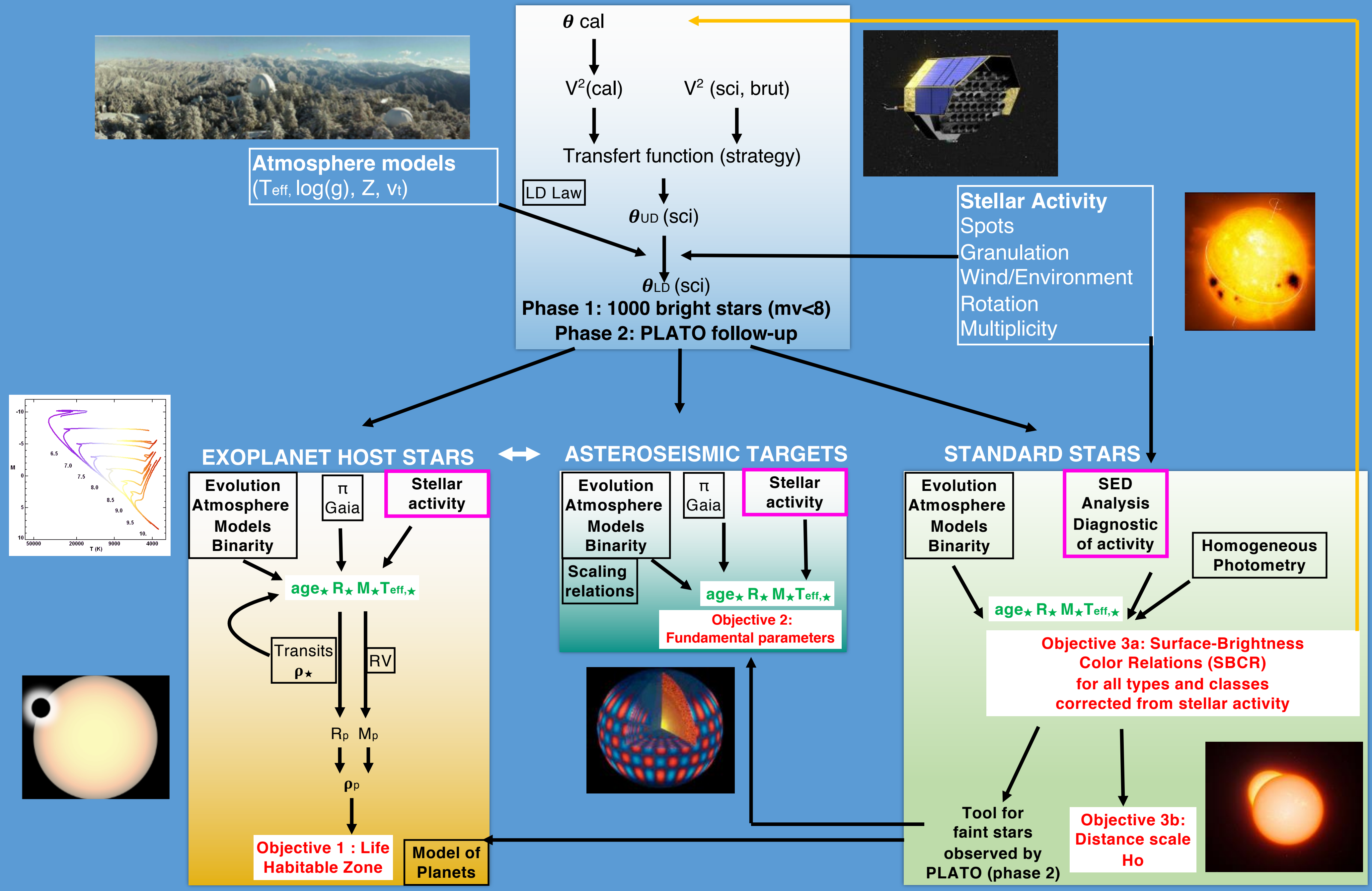
1. Exoplanet Host Stars



2. Asteroseismology



3. SBCR: for the distance of the eclipsing binaries and PLATO

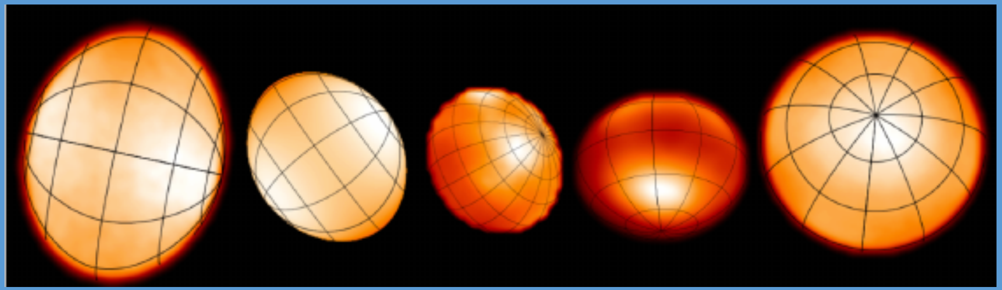

θ cal
V²(cal)
V² (sci, brut)
Transfert function (strategy)
LD Law
θUD (sci)
θLD (sci)
Phase 1: 1000 bright stars (mv<8)
Phase 2: PLATO follow-up
Atmosphere models
(Teff, log(g), Z, vt)
Stellar Activity
Spots
Granulation
Wind/Environment
Rotation
Multiplicity
EXOPLANET HOST STARS
Evolution Atmosphere Models Binarity
π Gaia
Stellar activity
age⋆ R⋆ M⋆ Teff,⋆
Transits ρ⋆
RV
Rp Mp
ρp
Objective 1 : Life Habitable Zone
Model of Planets
ASTEROSEISMIC TARGETS
Evolution Atmosphere Models Binarity
π Gaia
Stellar activity
Scaling relations
age⋆ R⋆ M⋆ Teff,⋆
Objective 2: Fundamental parameters
STANDARD STARS
Evolution Atmosphere Models Binarity
SED Analysis Diagnostic of activity
Homogeneous Photometry
age⋆ R⋆ M⋆ Teff,⋆
Objective 3a: Surface-Brightness Color Relations (SBCR) for all types and classes corrected from stellar activity
Tool for faint stars observed by PLATO (phase 2)
Objective 3b: Distance scale Ho

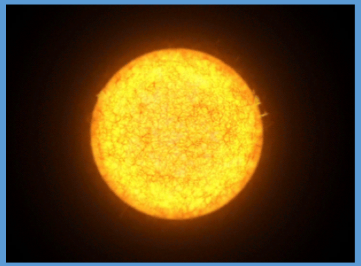
What means *large number* and *angular diameters* ?

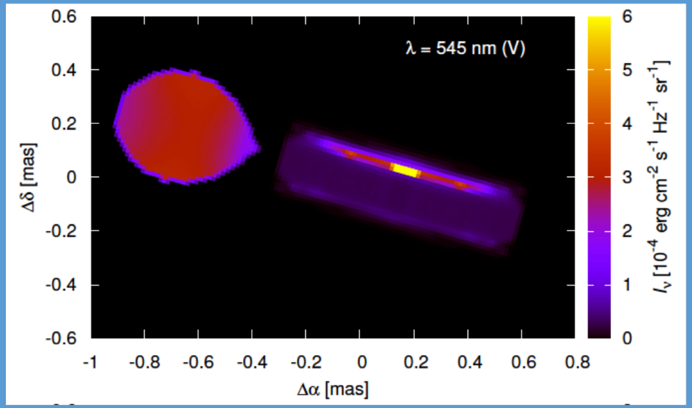
- Large Number:
  - In the past a few tens of objects only (PIONIER, CHARA)
  - For the SBC relations, 5 LC, 7SP ➔ few hundreds of stars for a good sampling of the HR diagram and to improve the precision and accuracy.
  - Almost 200 exoplanet host stars accessible to CHARA.

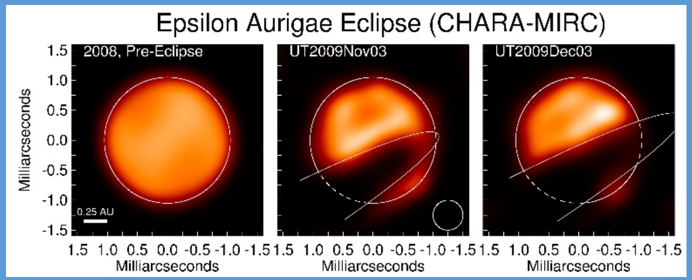
➔ ***~1000 stars***

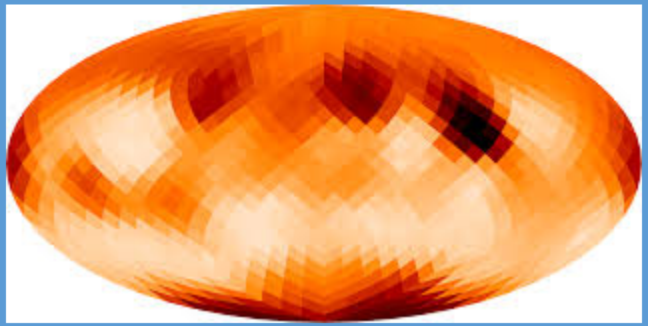
- Angular Diameters





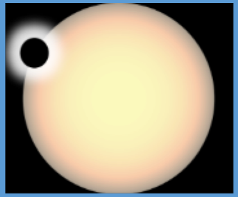
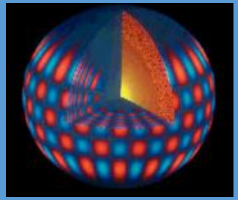






Three objectives:

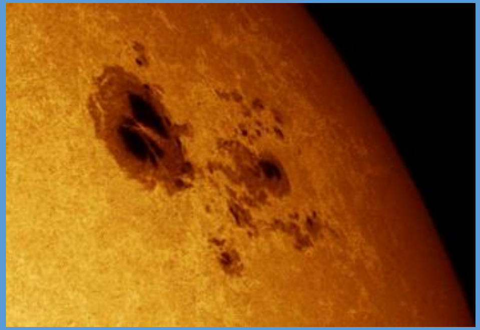
1. Exoplanet Host Stars
2. Asteroseismology
3. SBCR for distances of EB and PLATO


For these three objectives, stellar activity has to be taken into account:

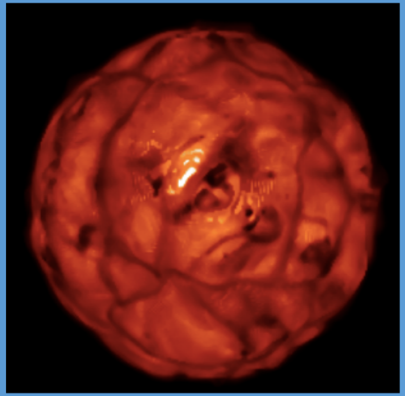
Stellar Activity

1 – Spots

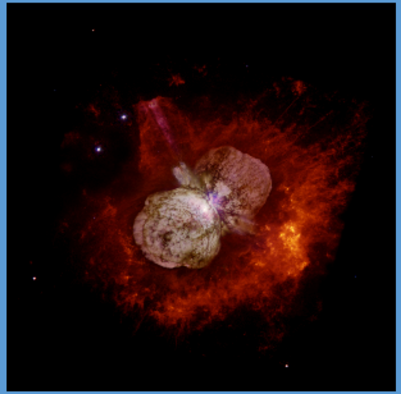


5 – Binarity

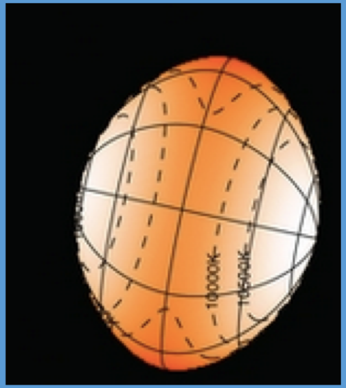
2 – Convection



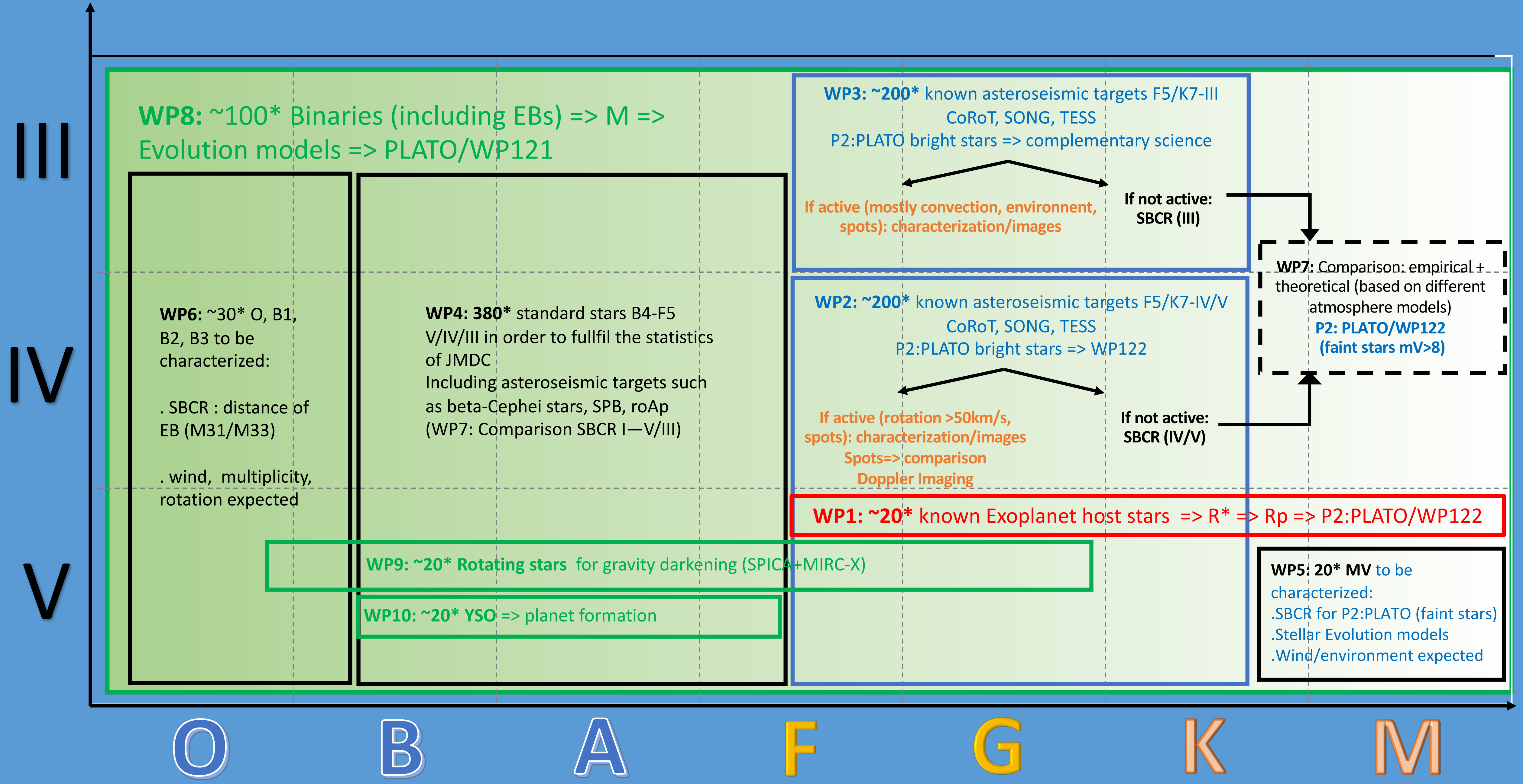
3 – Wind & environment



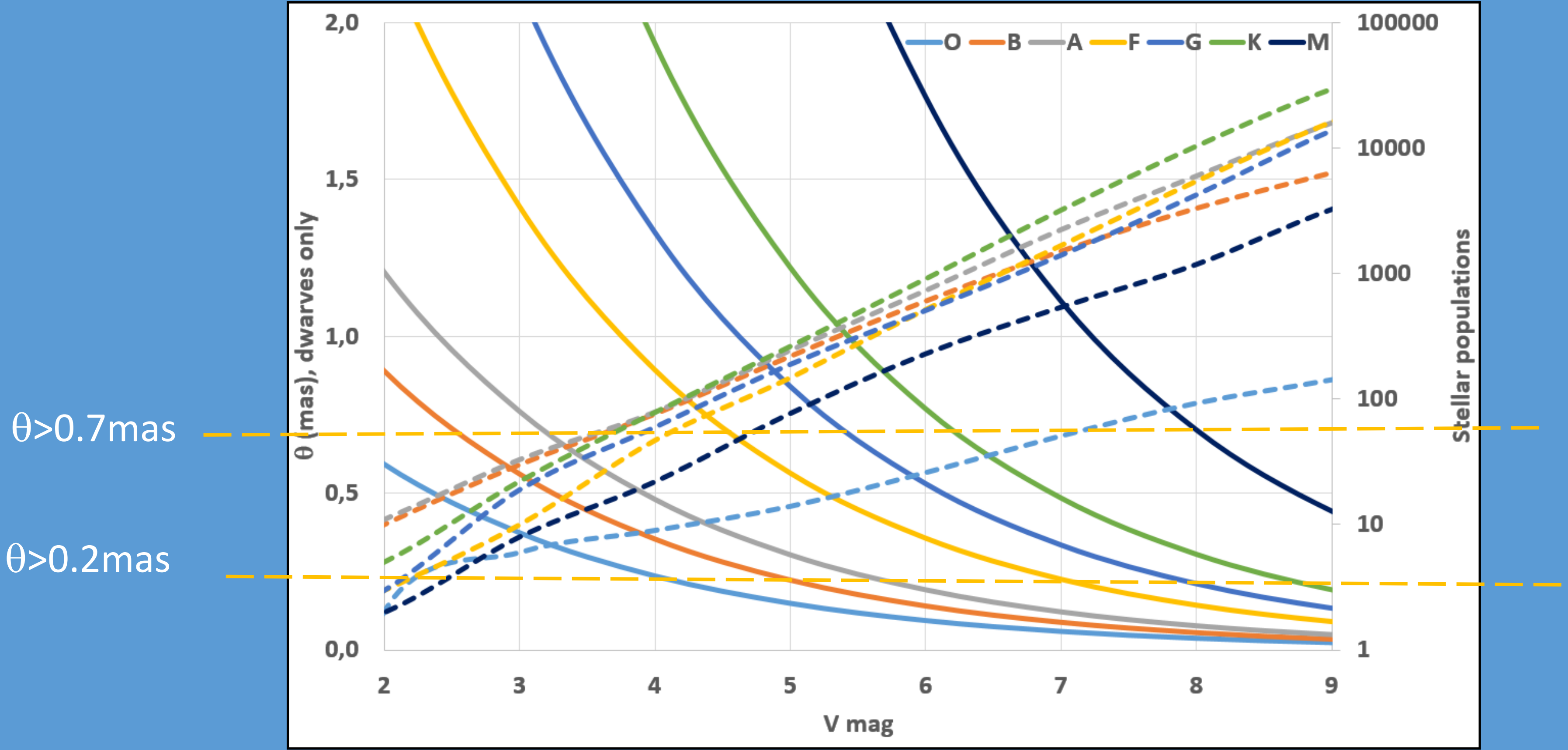
4 – Rotation



CHARA/SPICA phase 1 (2021=>2024): 800* standards (diameters) + 200* actives (images)



Statistics of the CHARA sky



	O	B	A	F	G	K	M
Limiting magnitude for θ>0.2mas	4.3	5.2	5.9	7.2	8.2	8.8	10.6
Number of stars with θ>0.2mas	10	266	646	2128	5420	23904	8377
Limiting magnitude for θ>0.7mas	1.5	2.5	3.3	4.5	5.4	6.2	8.0
Number of stars with θ>0.7mas	0	19	40	86	277	1153	1168

High level requirements
Diameters:

- magR=8 (but at low V^2)
- High precision, high efficiency (6T)
- R=300 (LR mode)

Imaging

- magR=5 (but at low V^2)
- UV coverage (6T, +Supersynthesis)
- R=3000 (MR mode)

Main scientific requirements

~1000 stars

→ θ down to 0.2mas

→ 300m and visible wavelengths is mandatory

Magnitude around 8 for the angular diameter measurements, around 4-5 for the surface imaging

SNR considerations → long exposures are mandatory to reach the sensitivity

Limiting magnitude defined as S/N=10 per spectral channel in 10mn of integration

Group delay only (DIT=10ms)

	R=140	R=3000
V^2=0.25	8.7	5.4
V^2=0.01	5.5	2.3

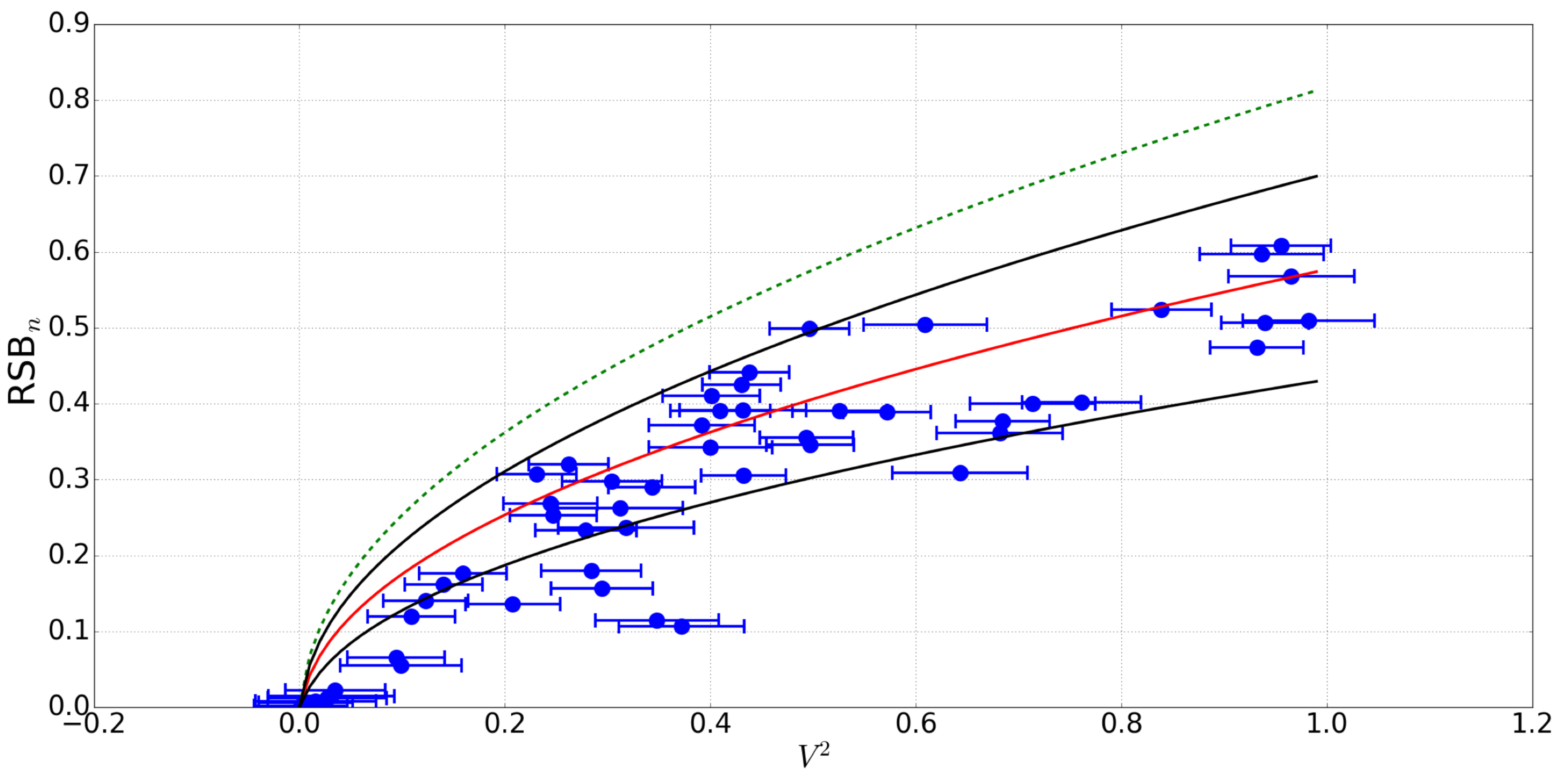
Phase delay tracking

	R=140	R=3000
V^2=0.25, DIT=0.2s	10.1	6.7
V^2=0.01, DIT=0.2s	6.7	3.5
V^2=0.25, DIT=30s	10.4	7.1
V^2=0.01, DIT=30s	7.0	4.0

These estimations use the same S/N calculator of FRIEND, validated on-sky

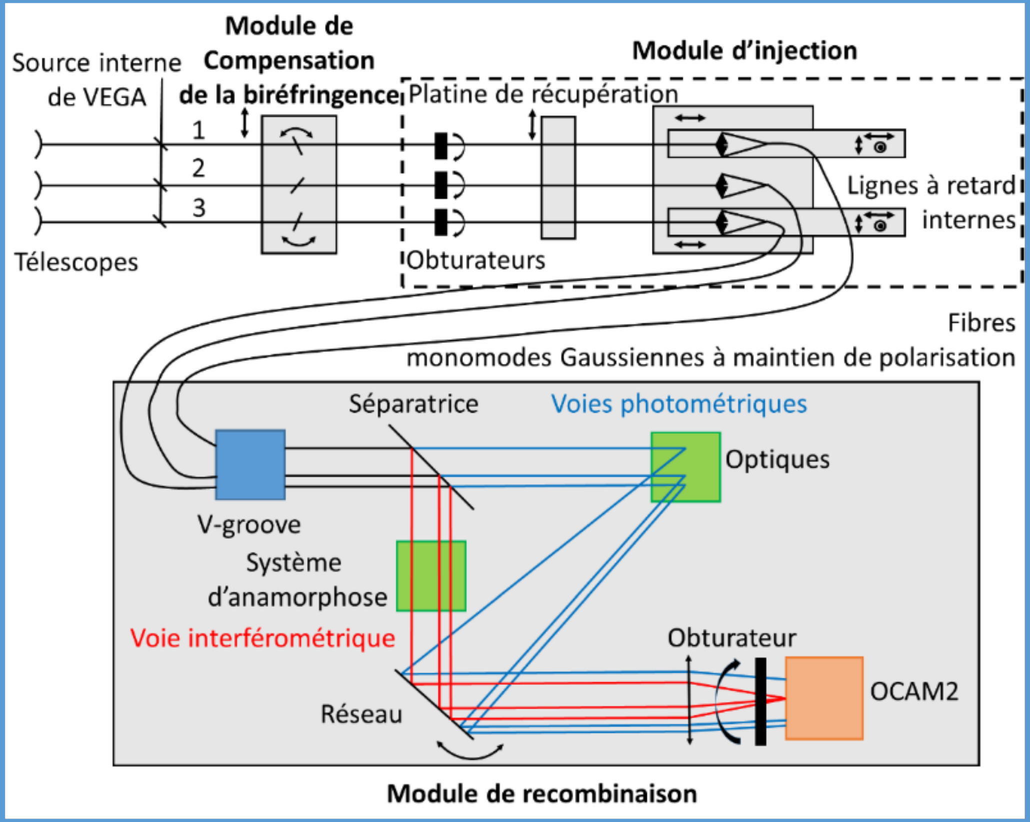
SNR model and sky calibration

$$RSB_{E_{HF},FRIEND} = \frac{\left(\frac{N_{ph}\,V_{instr}\,V_{obj}^{2}}{N_{tel}}\right)^{2}\ \widetilde{Gab_{ij}}\ \sqrt{N_{img}}}{\sqrt{\text{PhotonNoise} + \text{ReadNoise} + \text{CoupledTerms}}\ \sqrt{N_{\text{pic-frange}}}}$$



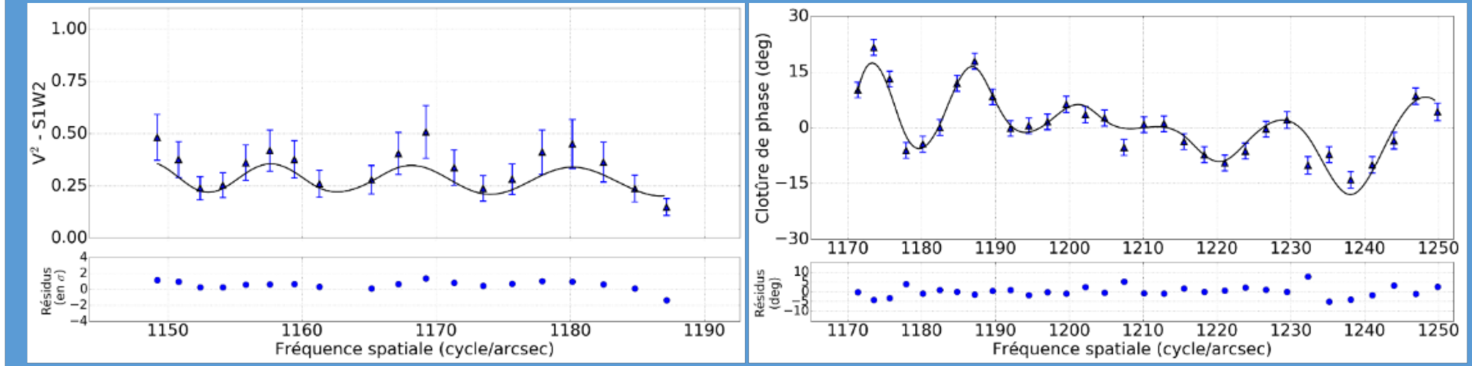
SPICA-VIS: The FRIEND prototype

Limitations of VEGA + AO on CHARA
➔ opportunity for fibered interferometry in the visible
➔ Prototype for know-how and expertise in Nice



Lessons learned on:

- Visible fibres and injection with partial AO
- Birefringence correction
- EMCCD detector
- Data processing with fibered combiner: V^2 and $C\phi$



Martinod et al., 2018

CHARA/SPICA Science Group Kick-off meeting (Jan. 2019)

(All presentations are online: https://chara-spica-ws.sciencesconf.org/)