The power-2 limb darkening law

Implementation and confrontation with observations

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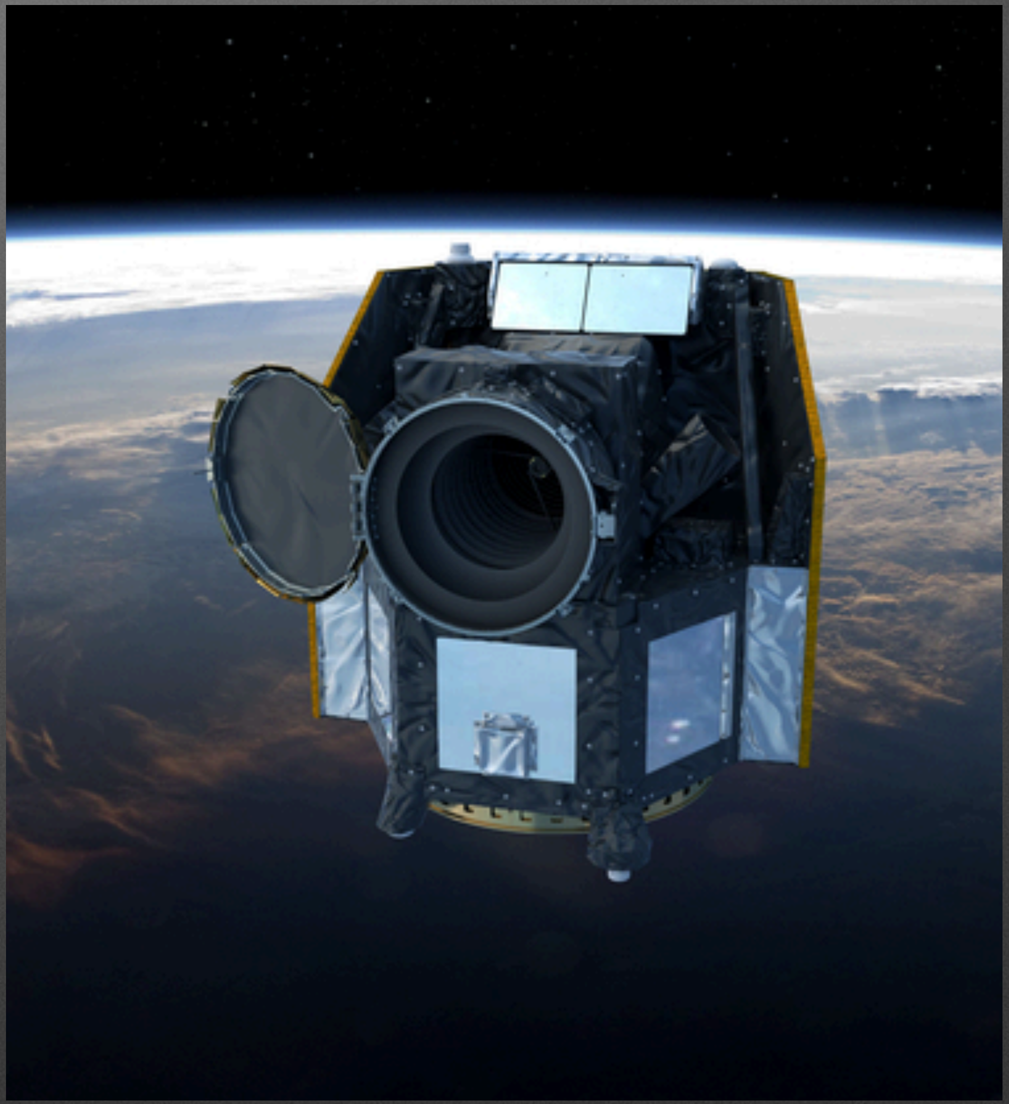
CHEOPS and pycheops

- ESA S-class mission to detect/ characterise exoplanet transits
- 32-cm telescope, low-Earth orbit, 400-1000nm bandpass
  - 150 ppm/min for V~9 star
- Launch 15th Oct — 14th Nov 2019.
- 20% of time for guest observers
- Open-source data analysis tools
  - *$ pip install pycheops*



Power-2 limb darkening law

$$I_\lambda (\mu) = 1 - c(1-\mu^\alpha)$$

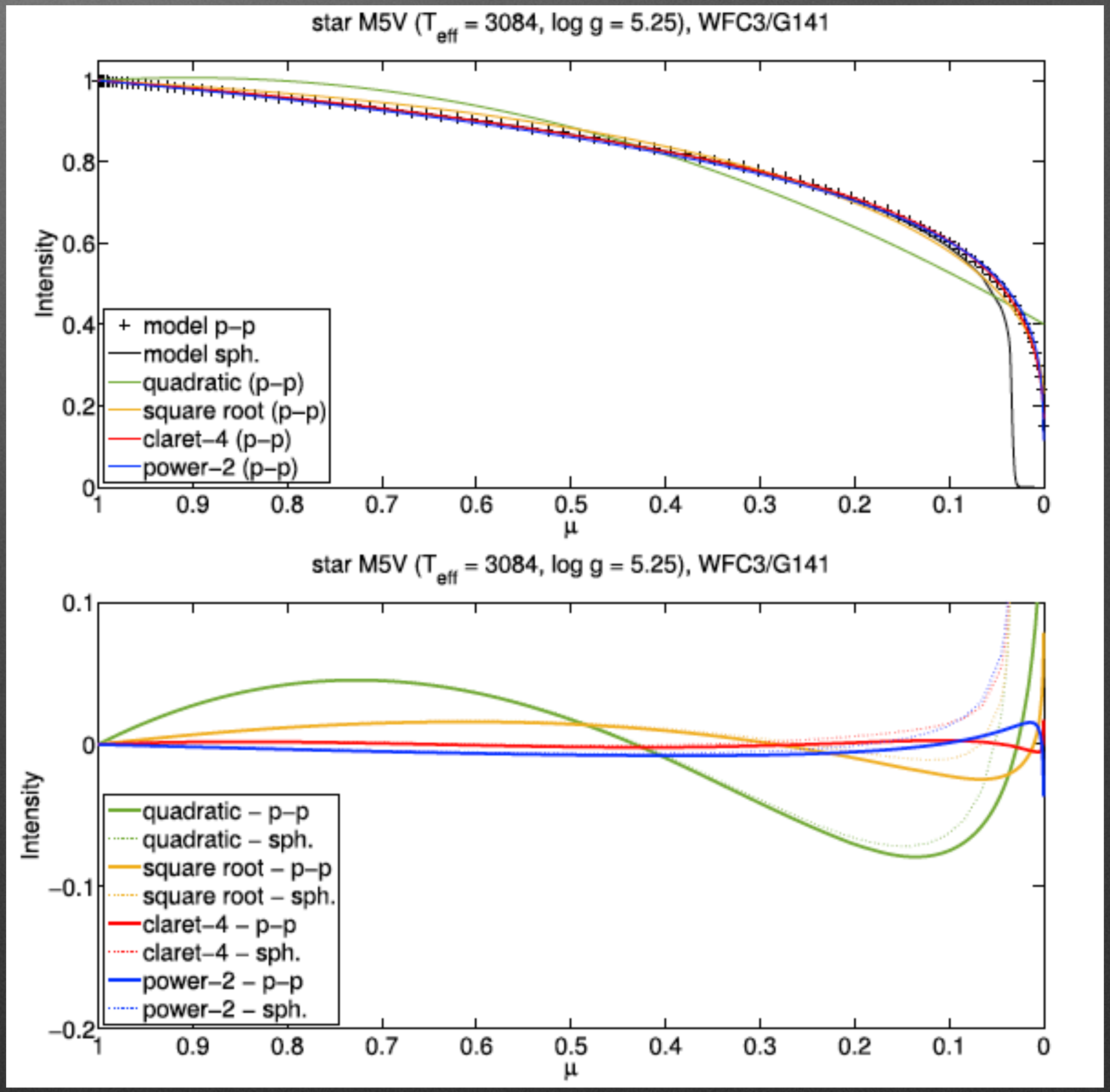
Morello et al. 2017

« recommend the use of …

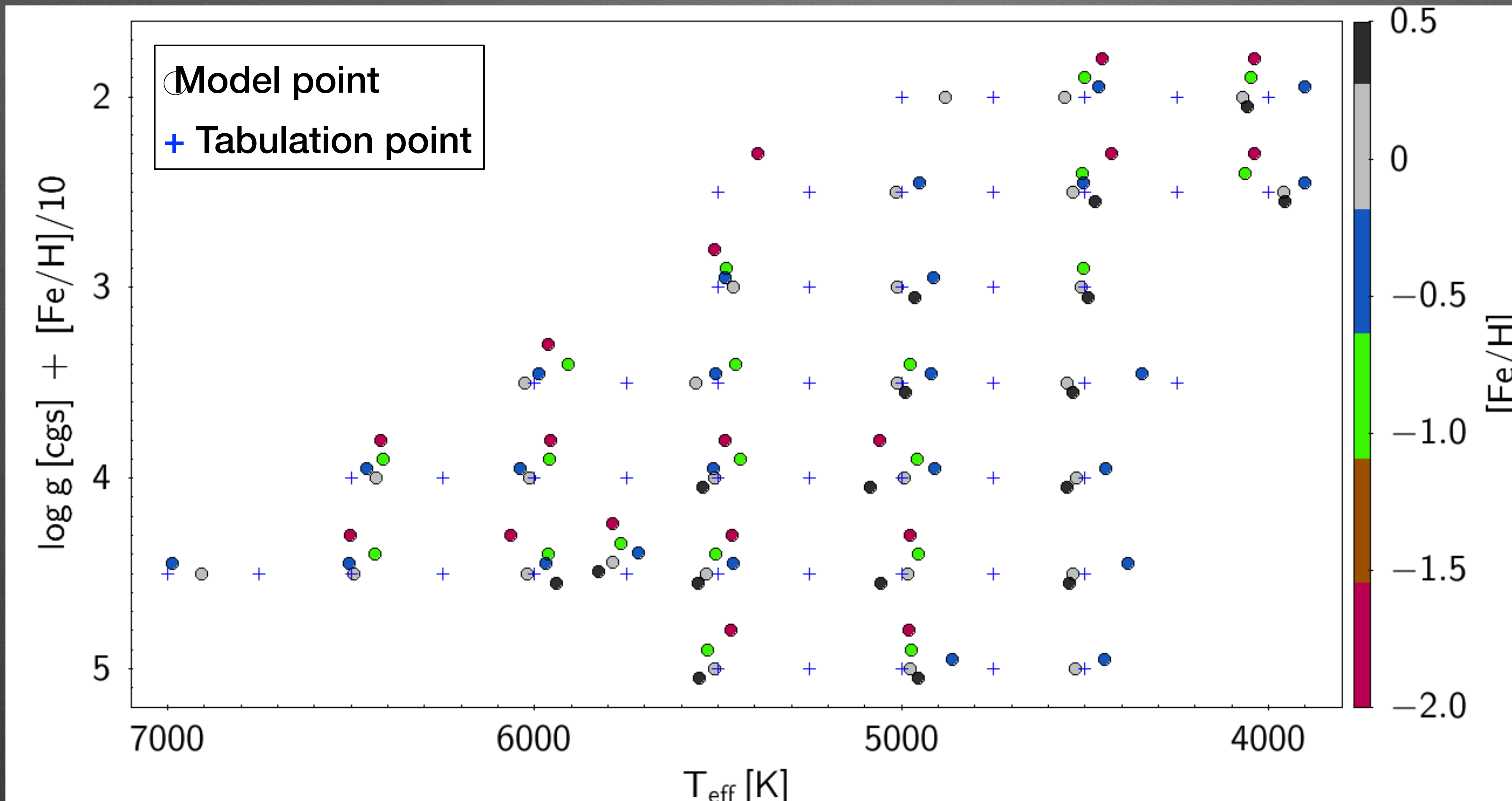
"power-2", which outperforms

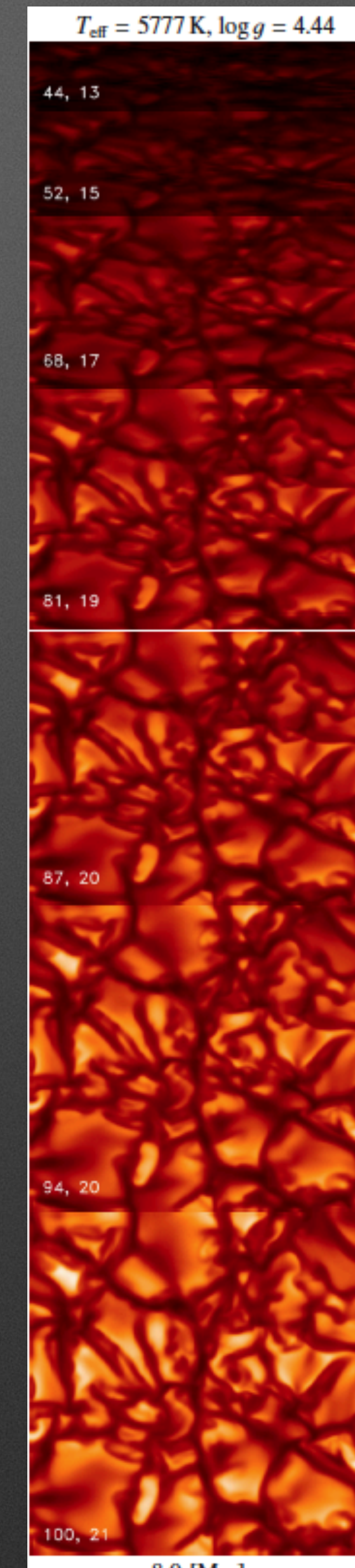
other two-coefficient laws …

particularly for cool stars »



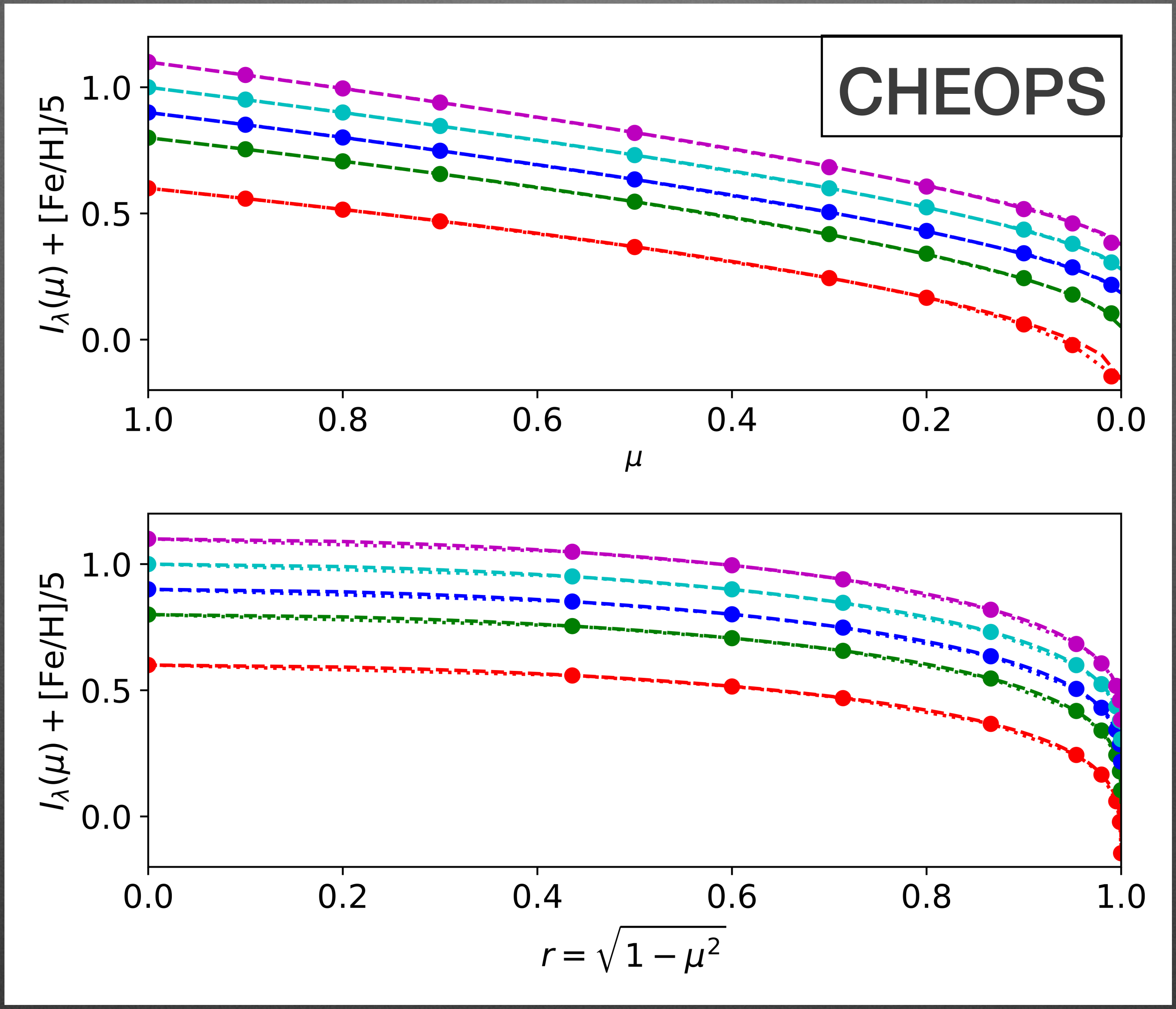
STAGGER-grid 3D RHD models





- mid-F to mid-K dwarf stars
- late-G to late-K giant stars

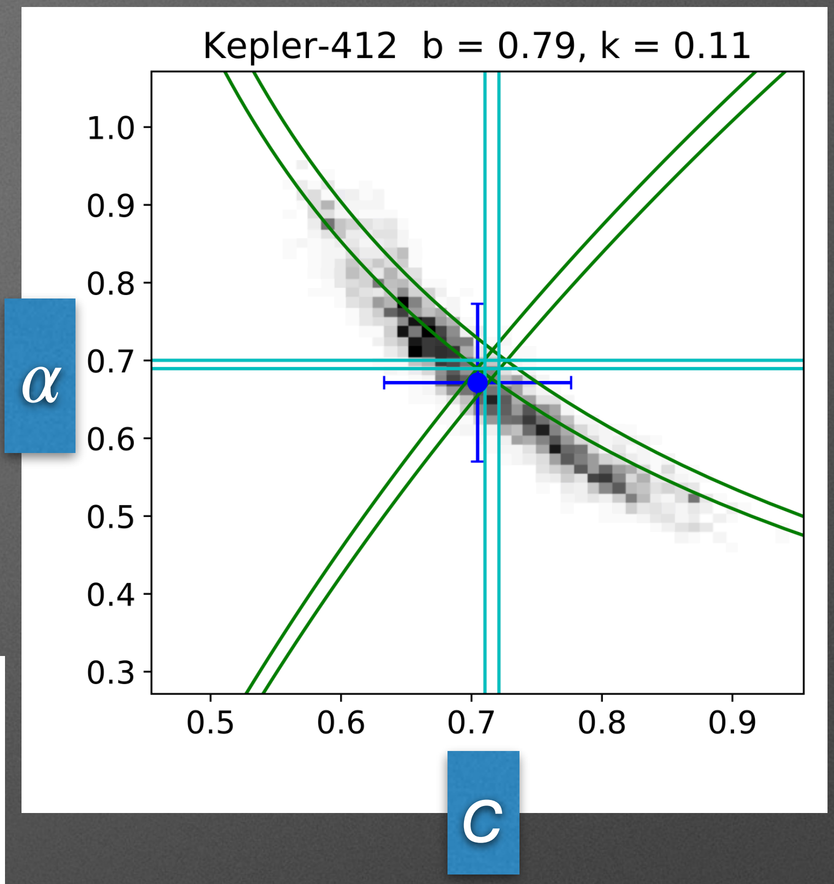
Putting power-2 into pycheops

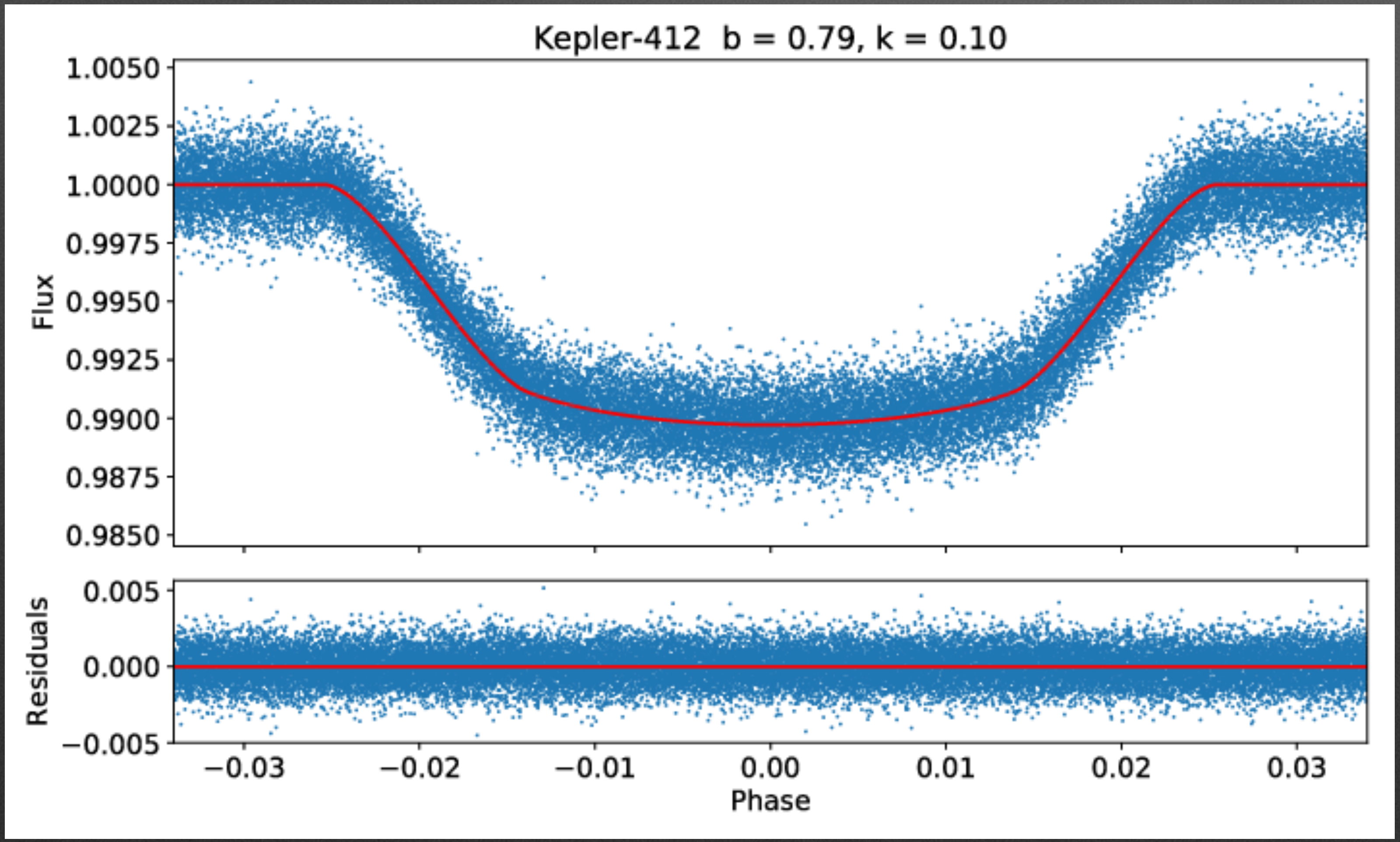


- Points computed directly from models
  - T_{eff}=5777, log g= 4.44, [Fe/H] = -2, -1, -0.5, 0.0, +0.5
- Dotted lines are interpolated profiles
- Dashed lines show power-2 law for interpolated c, α values

Observed power-2 law parameters

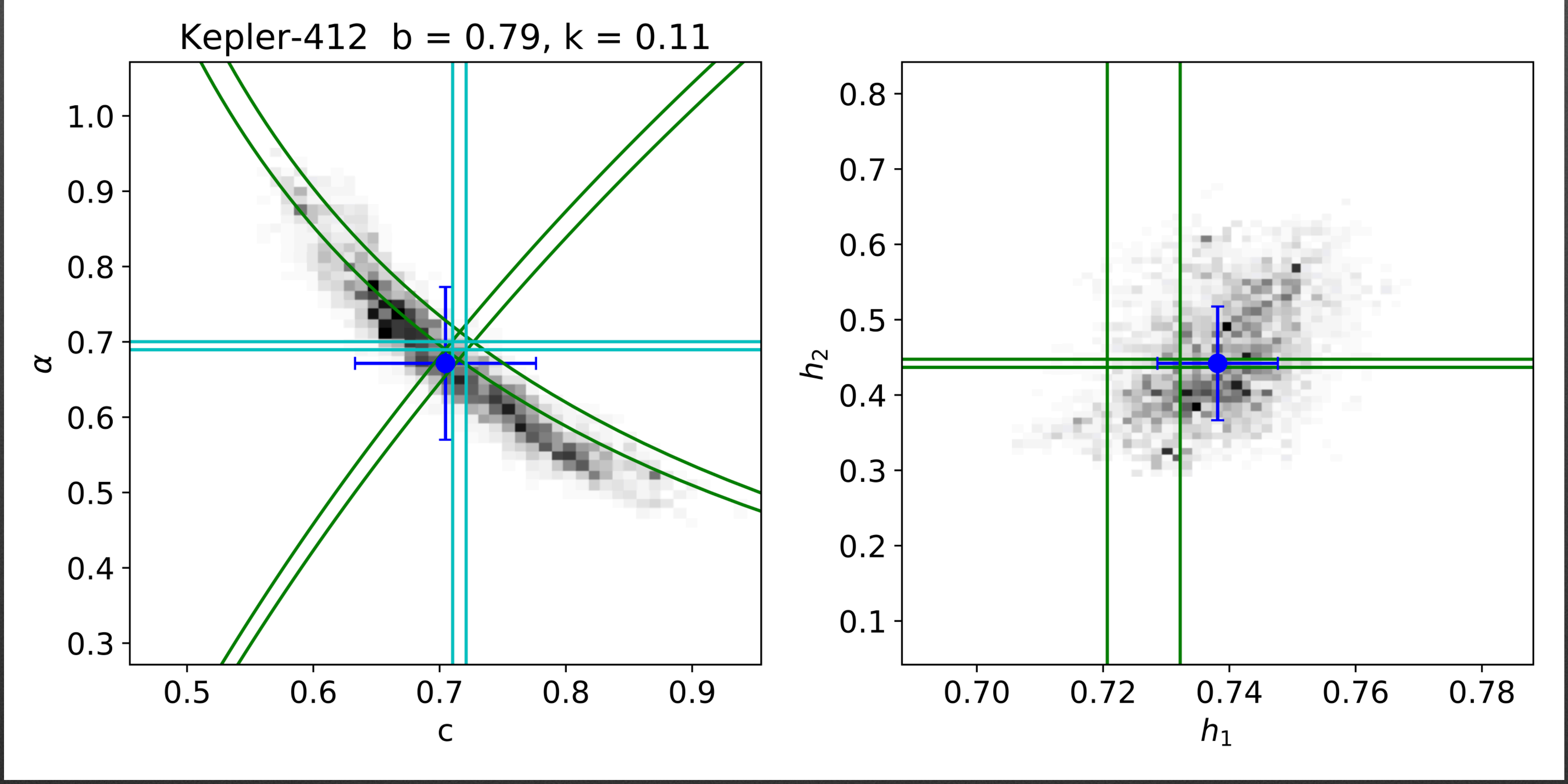
- ➡ Kepler DR25 SC light curves for transiting exoplanets
- ➡ Model with `ellc`
- ➡ Include c and α as free parameters



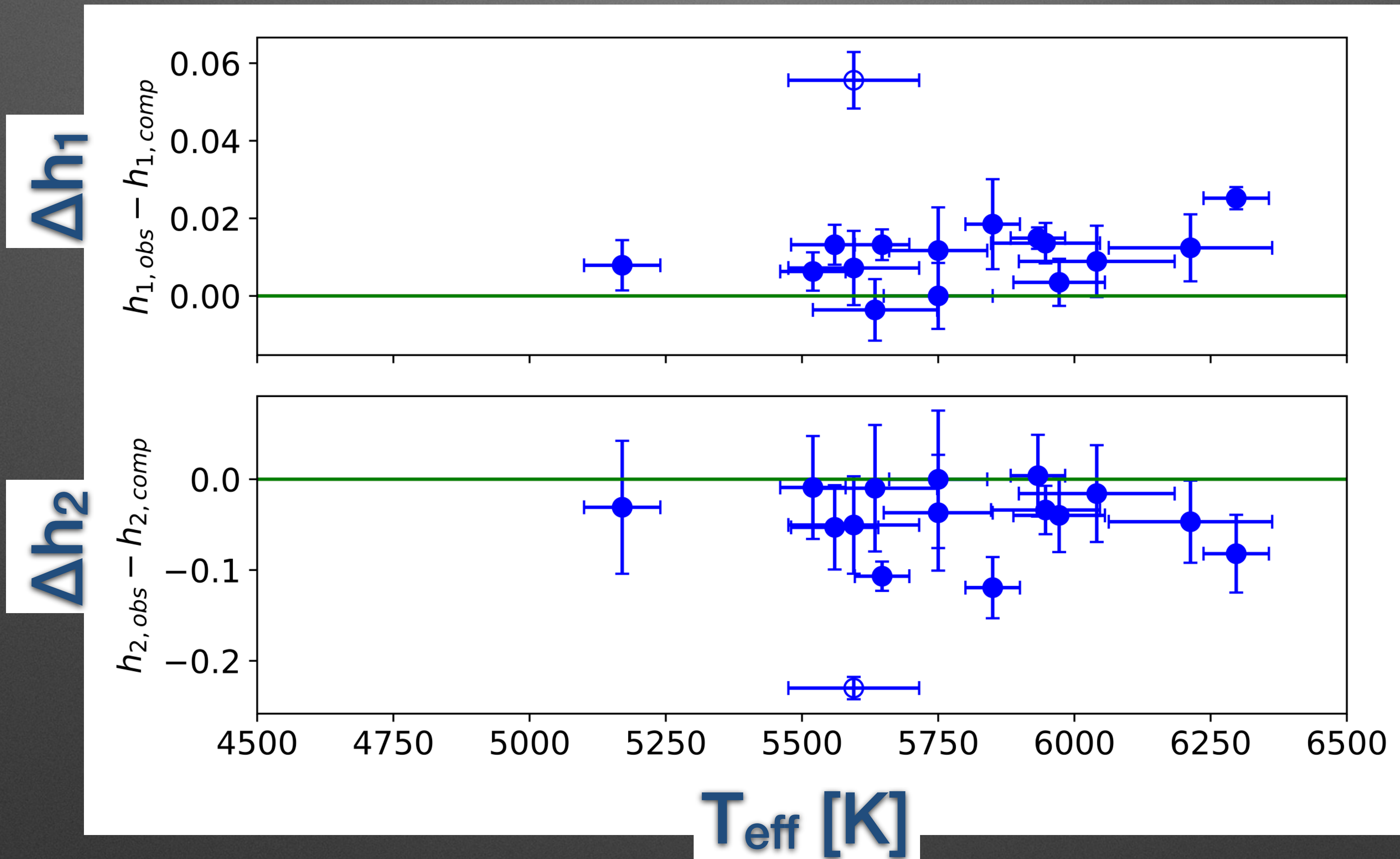


Comparing theory and observations

• $h_1 = I_\lambda(½)$ • $h_2 = I_\lambda(½) - I_\lambda(0)$	• $h_1 = 1 - c\,(1 - ½^\alpha)$ • $h_2 = ½^\alpha\, c$



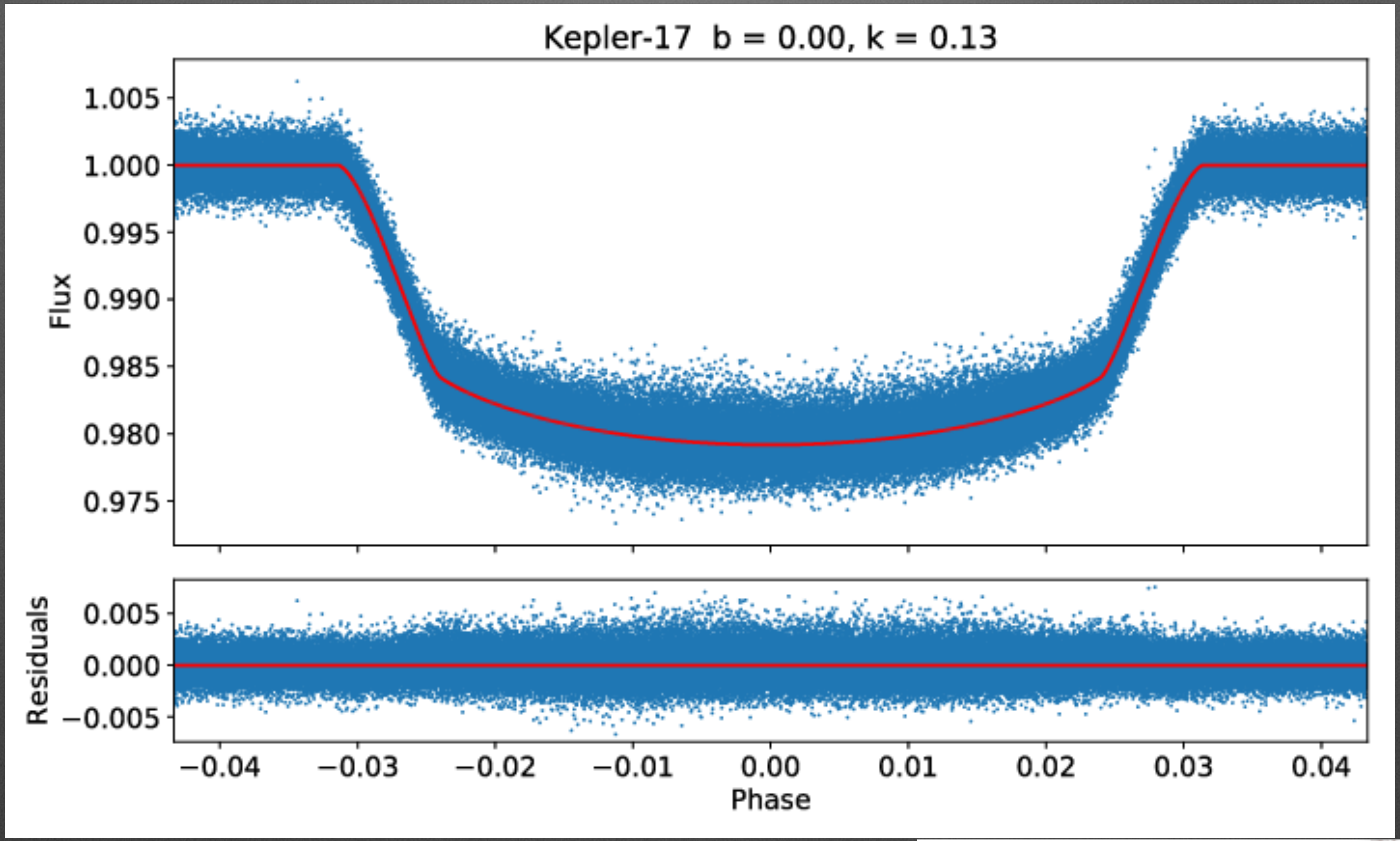
Power-2 law — model v. observations



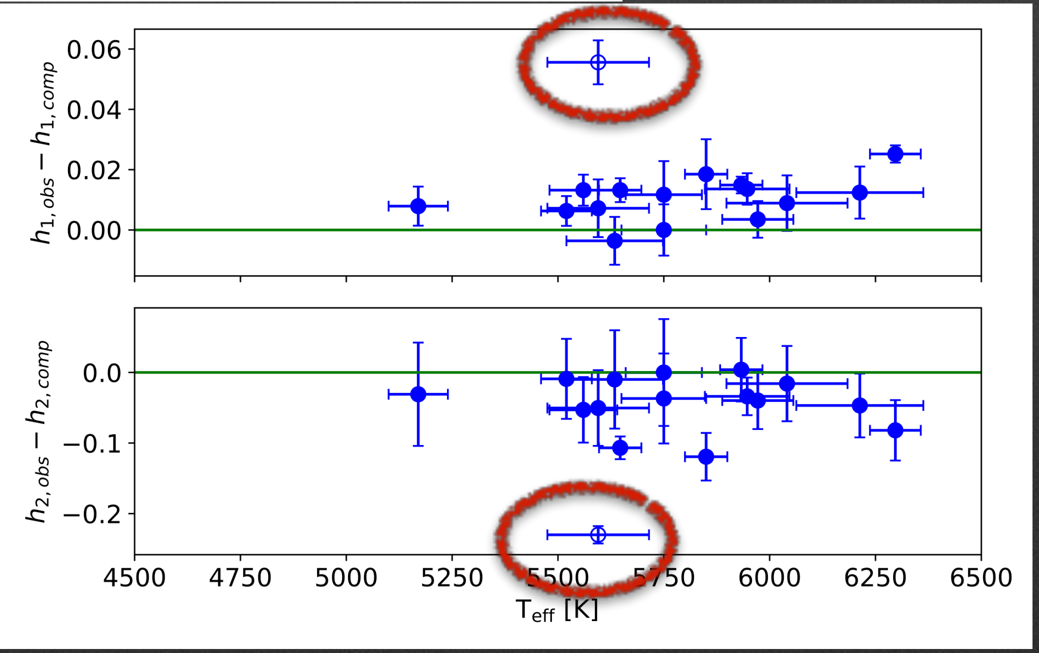
$\langle \Delta h_1 \rangle = +0.010 \pm 0.002 \ (\sigma = 0.011)$

$\langle \Delta h_2 \rangle = -0.042 \pm 0.010 \ (\sigma = 0.045)$

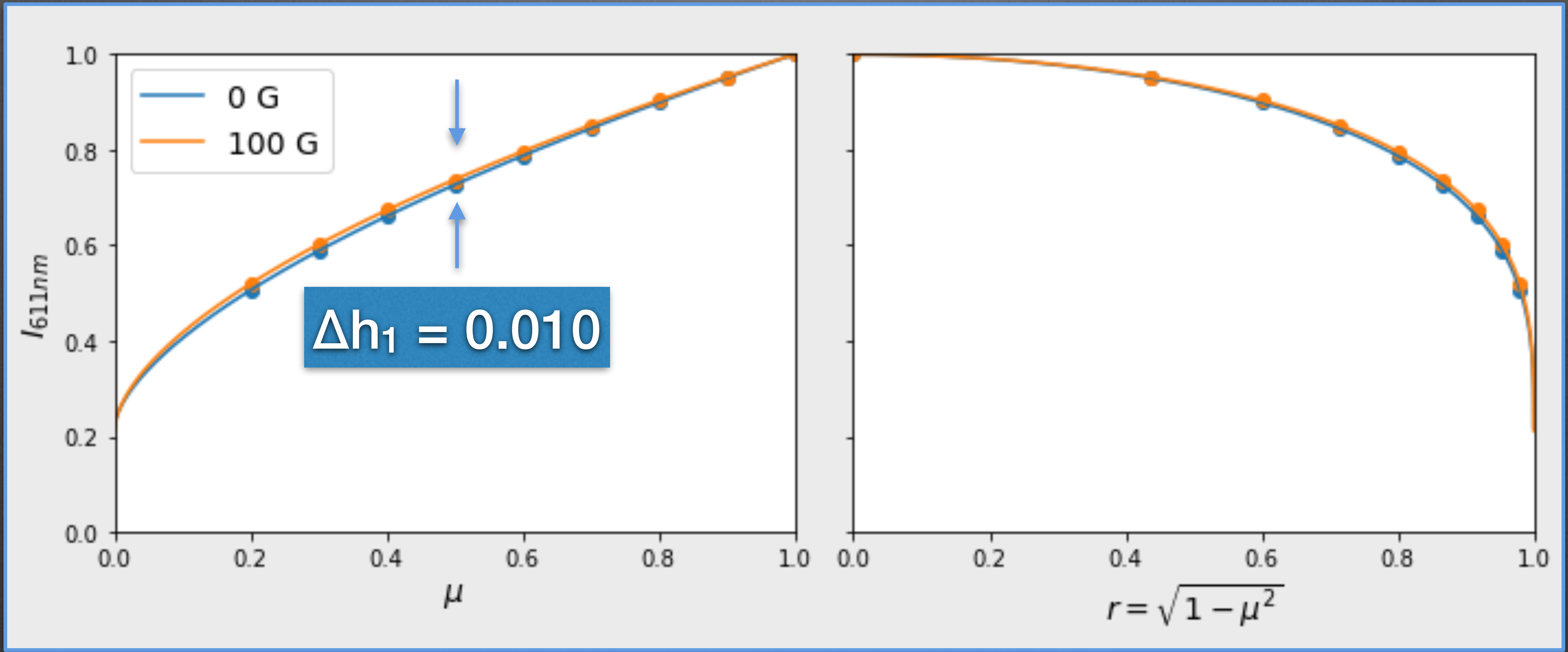
Power-2 law — model v. observations



Kepler-17 is an outlier.
Also the only star in the sample with clear star-spot activity.



Limb darkening in MURaM models



MURaM solar models, limb-darkening profiles at 611nm for 0G and with faculae at 100G (Norris et al., 2017)

N.B. These models not suitable for calculating h_2

Light curve models with power-2 limb-darkening

- ellc
  - *also does spots, Roche geometry, Doppler boosting, etc.*
- batman
- pycheops
  - *qpower2 algorithm*

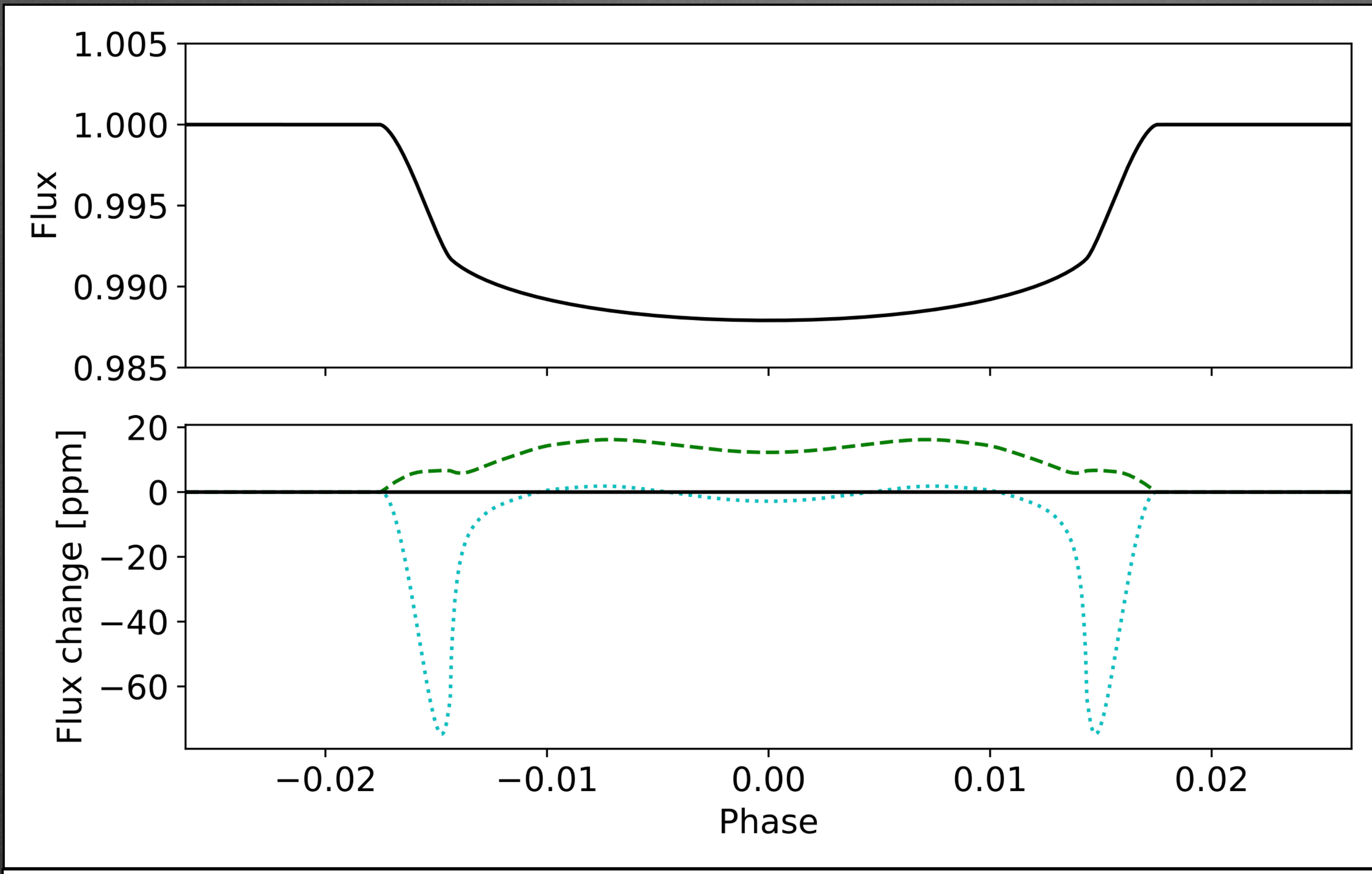
qpower2

- Fast power-2 light curve algorithm
- Based on Taylor series expansions of integrals
- Approximation
  - accurate to ~80ppm for $R_{planet}/R_{star}=0.1$

qpower2.py 〉 No Selection

```
def qpower2(z,p,c,alpha):
from numpy import arccos, sqrt, pi, clip, select, finfo
I_0 = (alpha+2)/(pi*(alpha-c*alpha+2))
g = 0.5*alpha
def q1(z,p,c,alpha):
zt = clip(abs(z), 0,1-p)
s = 1-zt**2
c0 = (1-c+c*s**g)
c2 = 0.5*alpha*c*s**(g-2)*((alpha-1)*zt**2-1)
return 1-I_0*pi*p**2*(c0 + 0.25*p**2*c2 - 0.125*alpha*c*p**2*s**(g-1))
def q2(z,p,c,alpha):
zt = clip(abs(z), 1-p,1+p)
d = clip((zt**2 - p**2 + 1)/(2*zt),0,1)
ra = 0.5*(zt-p+d)
rb = 0.5*(1+d)
sa = clip(1-ra**2,finfo(0.0).eps,1)
sb = clip(1-rb**2,finfo(0.0).eps,1)
q = clip((zt-d)/p,-1,1)
w2 = p**2-(d-zt)**2
w = sqrt(clip(w2,finfo(0.0).eps,1))
b0 = 1 - c + c*sa**g
b1 = -alpha*c*ra*sa**(g-1)
b2 = 0.5*alpha*c*sa**(g-2)*((alpha-1)*ra**2-1)
a0 = b0 + b1*(zt-ra) + b2*(zt-ra)**2
a1 = b1+2*b2*(zt-ra)
aq = arccos(q)
J1 = ( (a0*(d-zt)-(2/3)*a1*w2 + 0.25*b2*(d-zt)*(2*(d-zt)**2-p**2))*w
+ (a0*p**2 + 0.25*b2*p**4)*aq )
J2 = alpha*c*sa**(g-1)*p**4*(0.125*aq +
(1/12)*q*(q**2-2.5)*sqrt(clip(1-q**2,0,1)) )
d0 = 1 - c + c*sb**g
d1 = -alpha*c*rb*sb**(g-1)
K1 = ((d0-rb*d1)*arccos(d) +
((rb*d+(2/3)*(1-d**2))*d1 - d*d0)*sqrt(clip(1-d**2,0,1)) )
K2 = (1/3)*c*alpha*sb**(g+0.5)*(1-d)
return 1 - I_0*(J1 - J2 + K1 - K2)
return select( [z <= (1-p), abs(z-1) < p],
[q1(z, p, c, alpha), q2(z, p, c, alpha)], default=1)
```

qpower2 - example



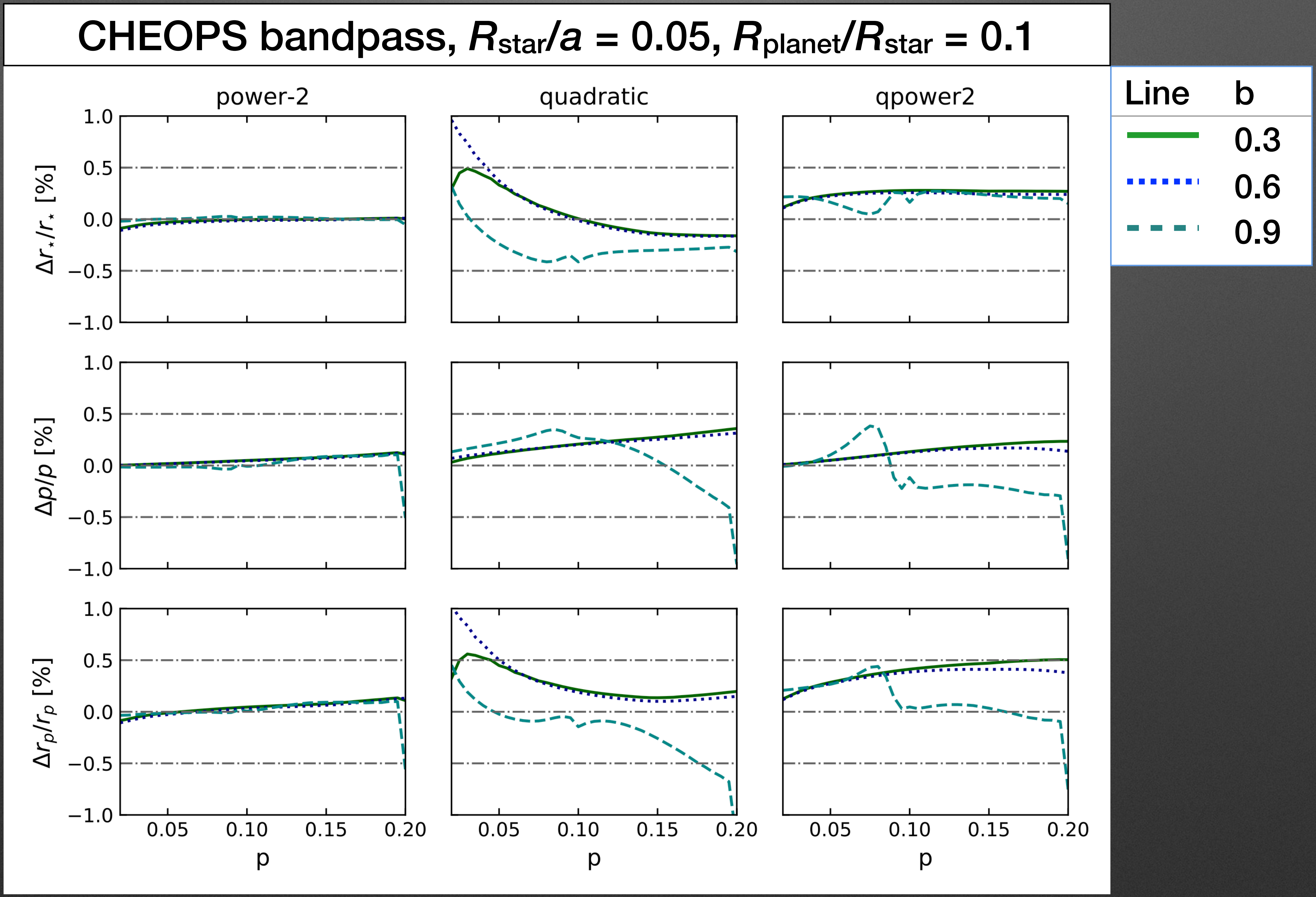
STAGGER-grid profile, ell

power-2 limb darkening, ell

qpower-2 algorithm

- CHEOPS band
- R_{star}/a = 0.05
- R_{planet}/R_{star} = 0.1
- b = 0
- T_{eff} = 6000 K
- log g = 4.5
- [Fe/H] = 0

qpower2 performance — accuracy



qpower2 performance— speed

Execution time for transit with 3840 observations

Method	Time per model
1 CPU	357.0 μs
8 CPUs	112.0 μs
GPU Batch	13.2 μs
GPU logL only	2.5 μs

⇒ 1 million *log L* values per second on GPUs

Conclusion

We should use the power-2 limb-darkening law to search for and to model transits in PLATO light curves of F/G/K-type dwarfs.

Resources, references

- **ellc binary star model**
  - Maxted, 2016 A&A 591, A111
  - ***$ pip install ellc***
- **pycheops**
  - ***$ pip install pycheops***
  - Documentation and Bayesian model fitting under development
- **Limb darkening profiles and power-2 parameters from STAGGER-grid**
  - Maxted, 2018, A&A 616, A39
  - Vizier: J/A+A/616/A39/table2
- **qpower2**
  - Maxted & Gill, 2019, A&A, 622 A33
  - ***$ pip install pycheops***