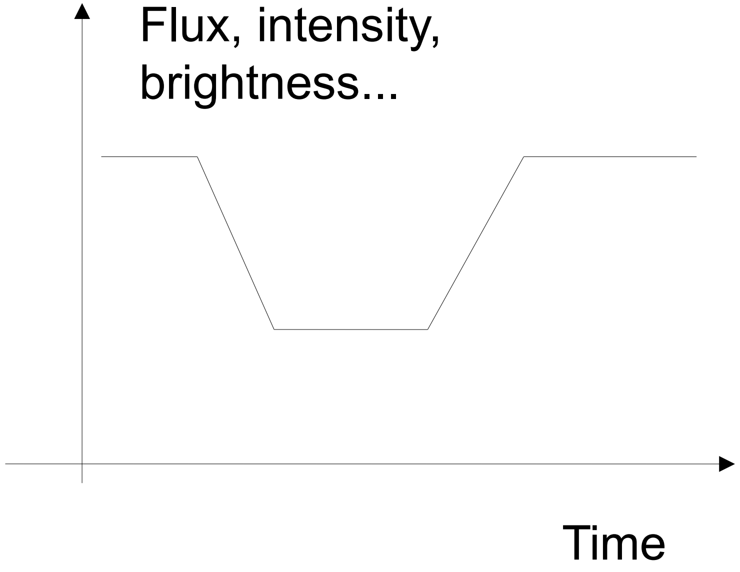
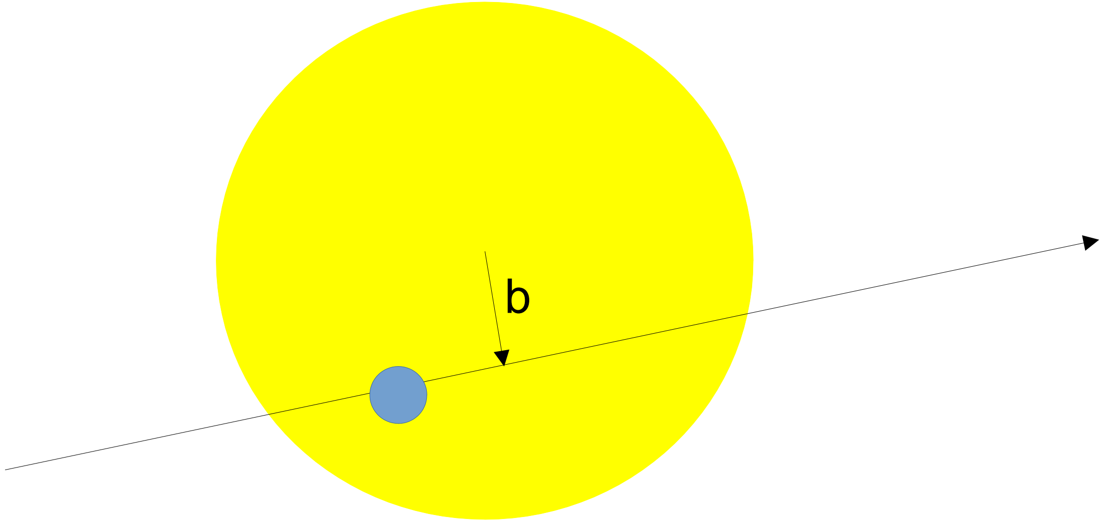
The role of limb darkening in the analysis of exoplanetary transit light curves

Szilárd Csizmadia (WP 114 000: Transit Tools)

Granada, Spain, 26-27 February 2019

PLATO WP 122 Limb Darkening Workshop

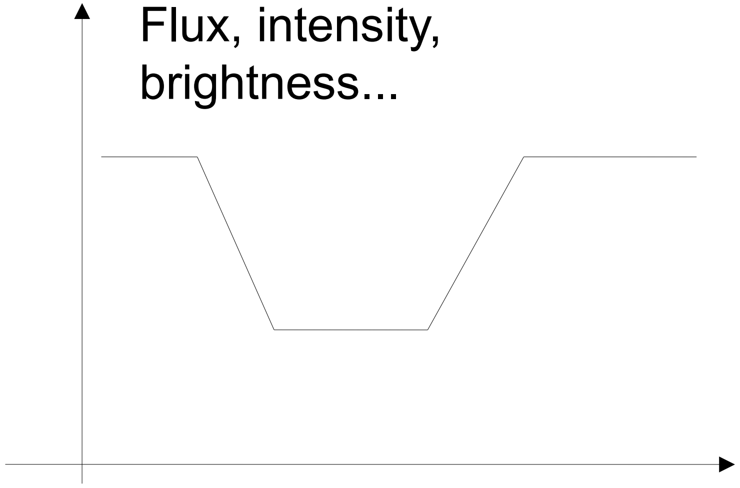


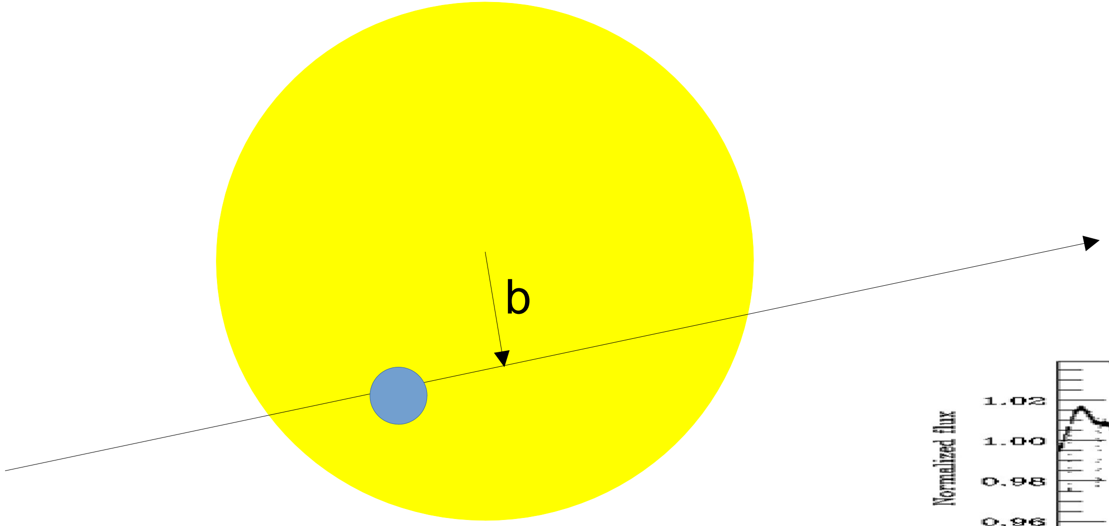


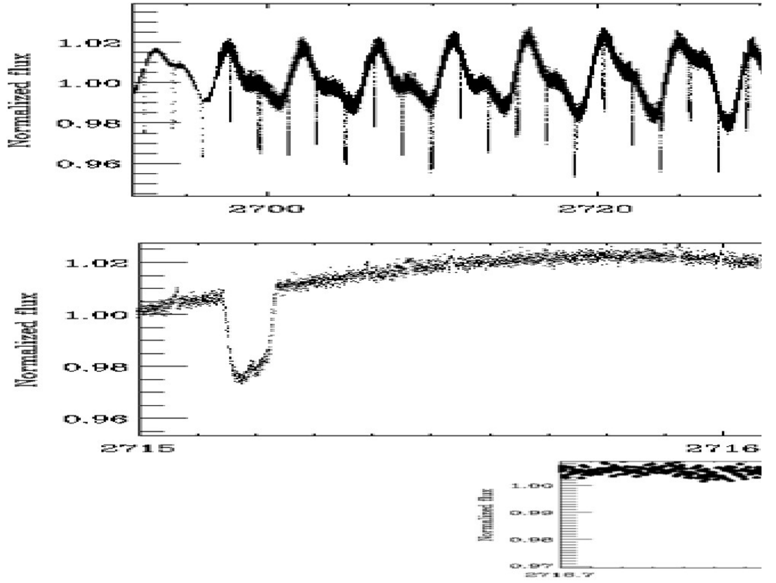
$$\frac{\Delta F}{F} \approx \left(\frac{R_{planet}}{R_{star}} \right)^2$$

b: impact parameter

PLATO’s goal: 3% in R_{planet}, 2% in R_{planet}/R_{star}.



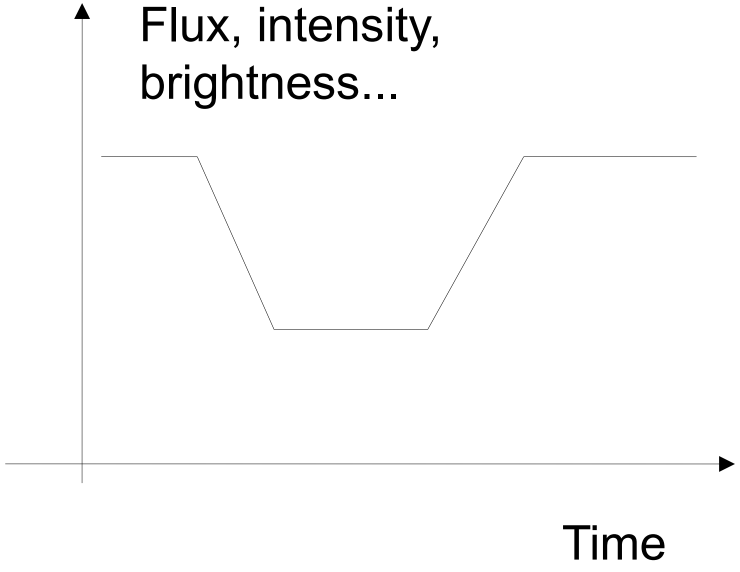


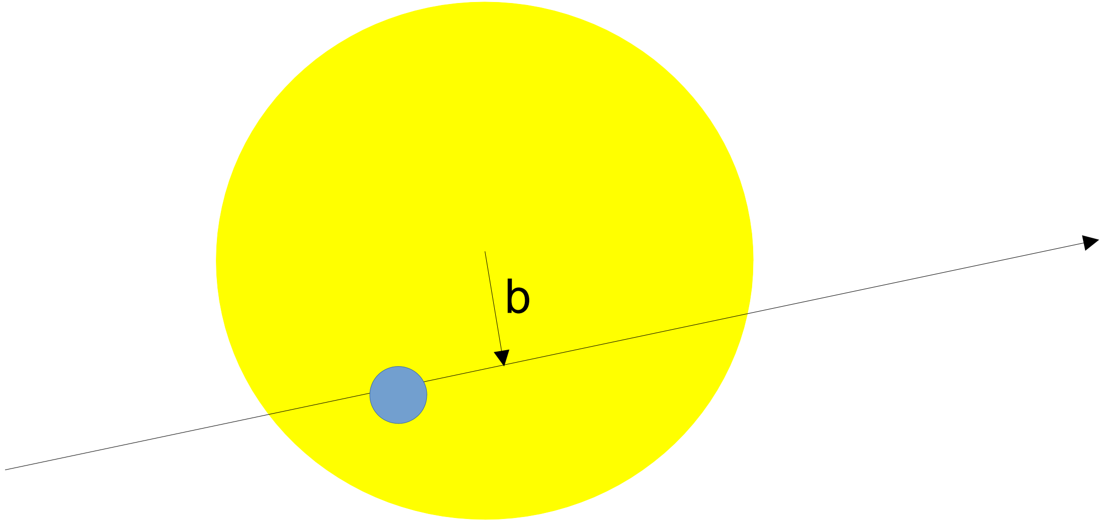


$$\frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}} \right)^2$$

Baseline variations (instrumental + stellar activity)
Precision issues, systematic flux-shifts, ...

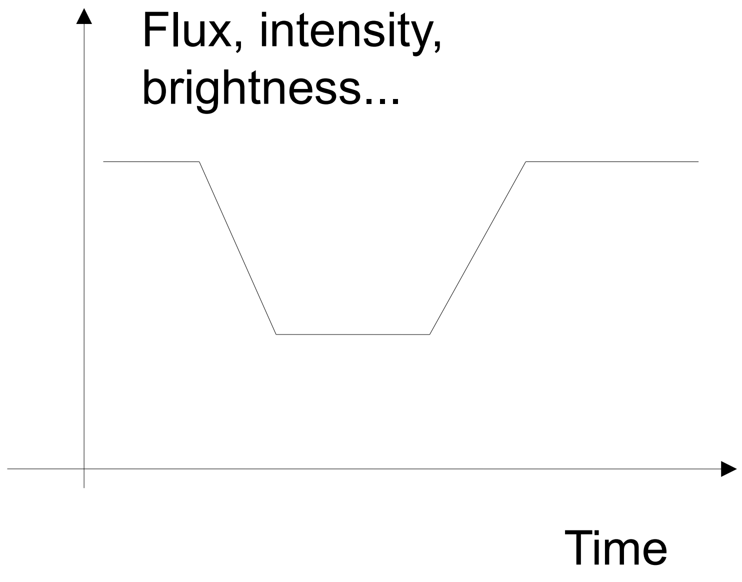
CoRoT-2b
Alonso et al.
(2009)

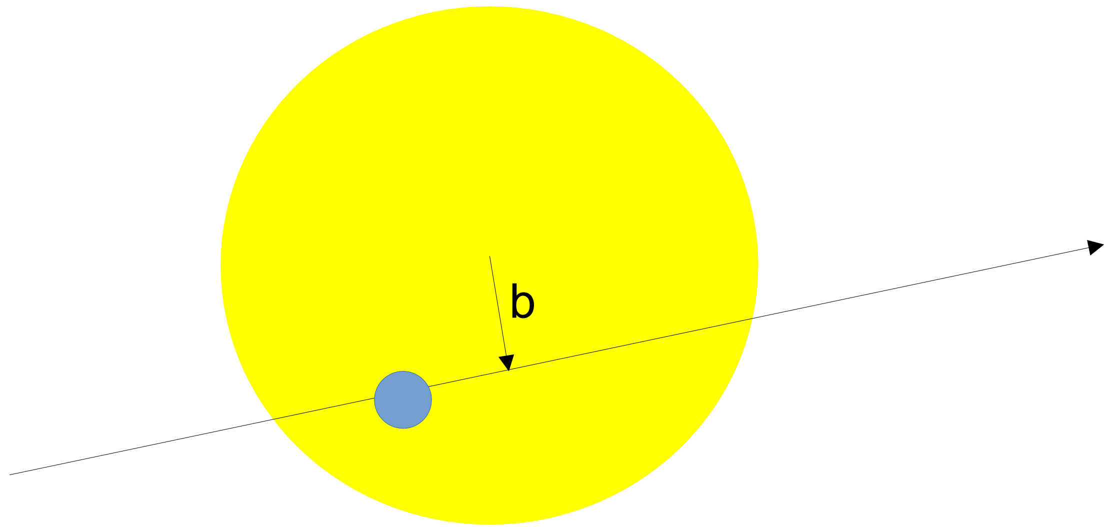




$$\frac{\Delta F}{F}=\left(\frac{R_{planet}}{R_{star}}\right)^2$$

Short time scale variations (instrumental, stellar spots, granulation, flares, ...)
Precision issues, linearity of the CCD...

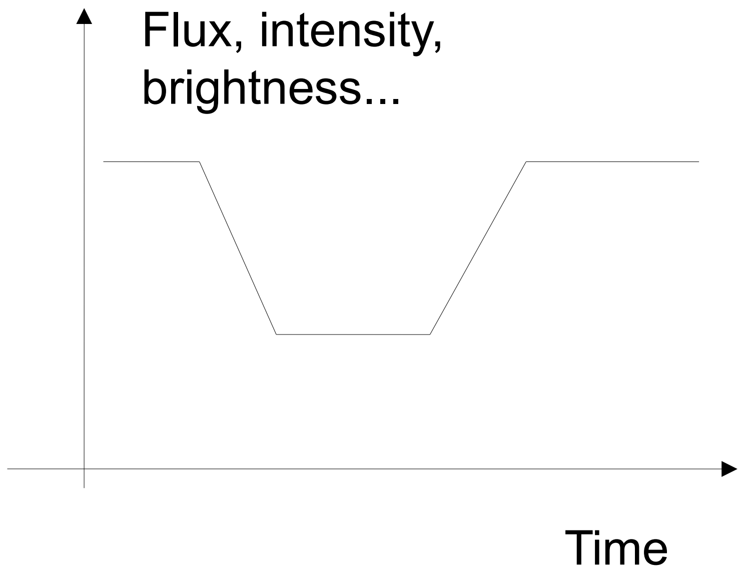


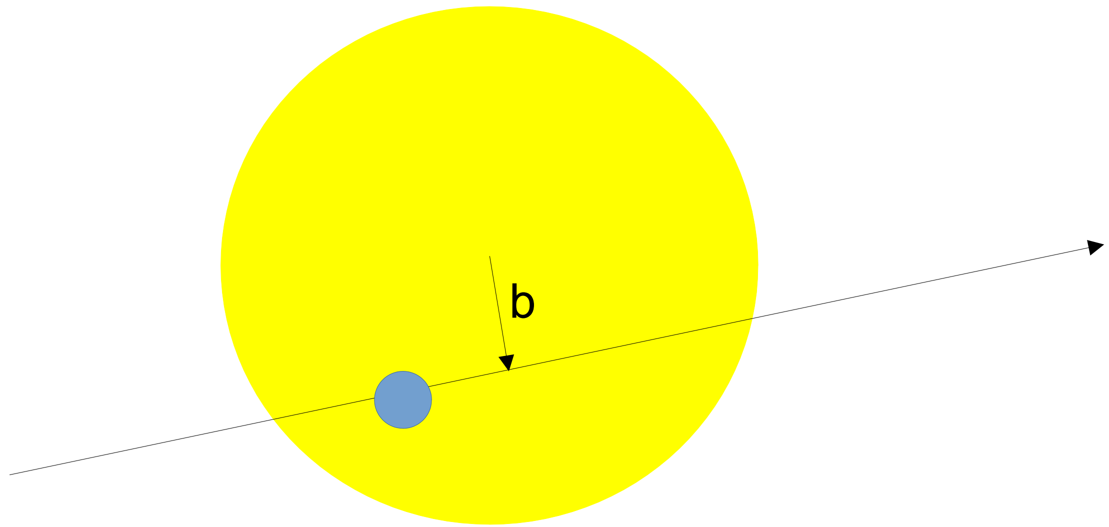


$$\frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}} \right)^2$$

Will be known to 1% accuracy and precision.

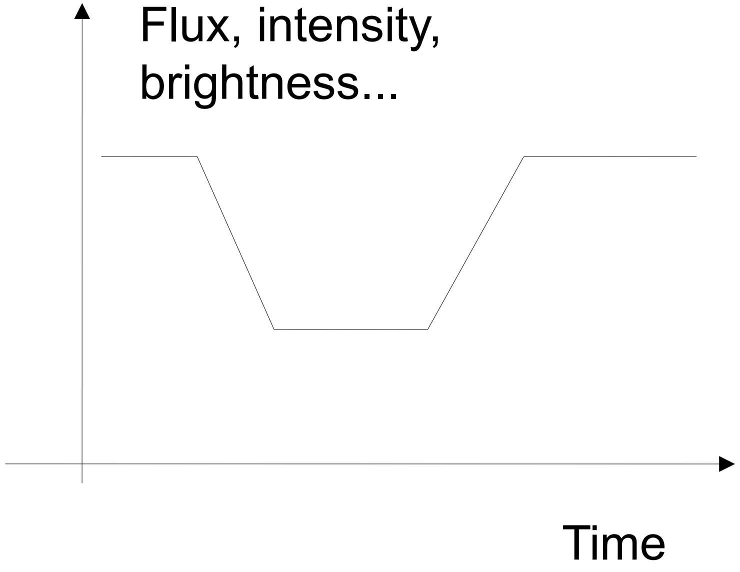
DLR

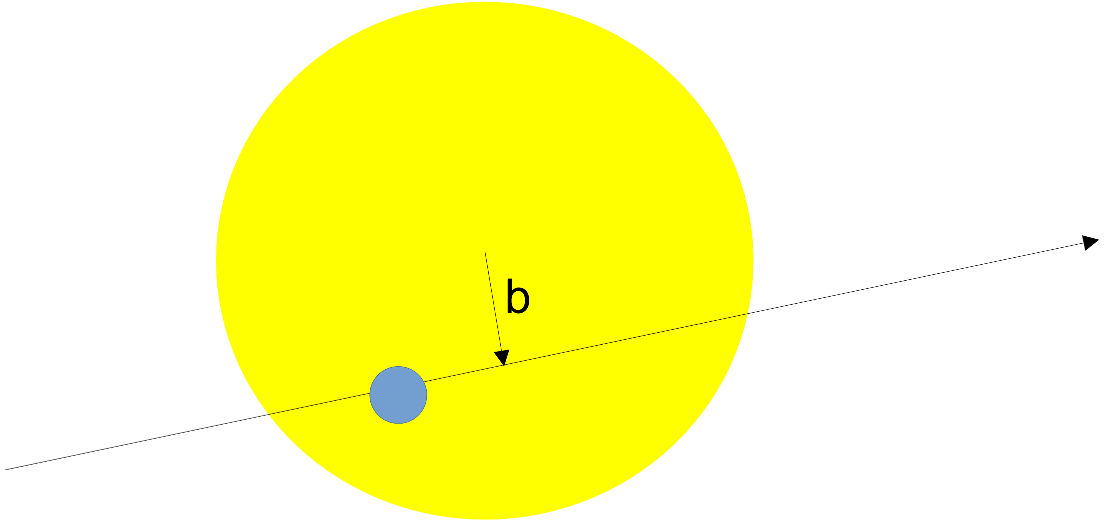




$$\frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}} \right)^2$$

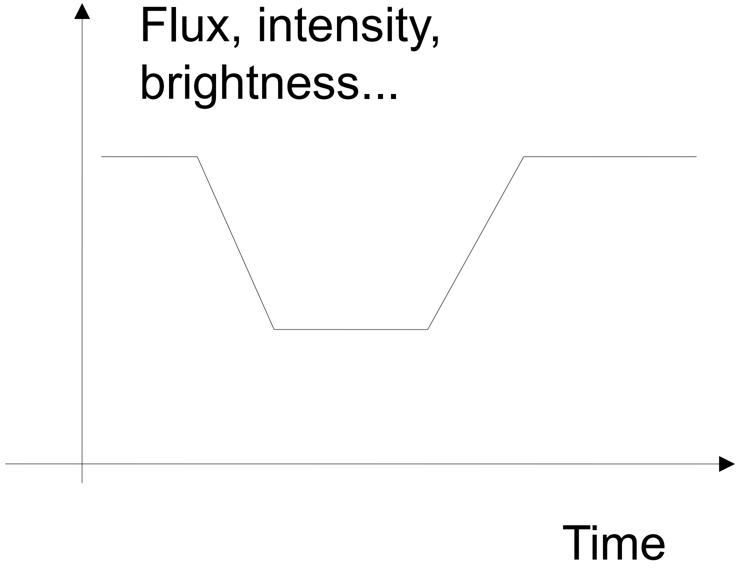
Assumed to be spherical, absolutely dark (no nightside emission), no atmosphere, ...

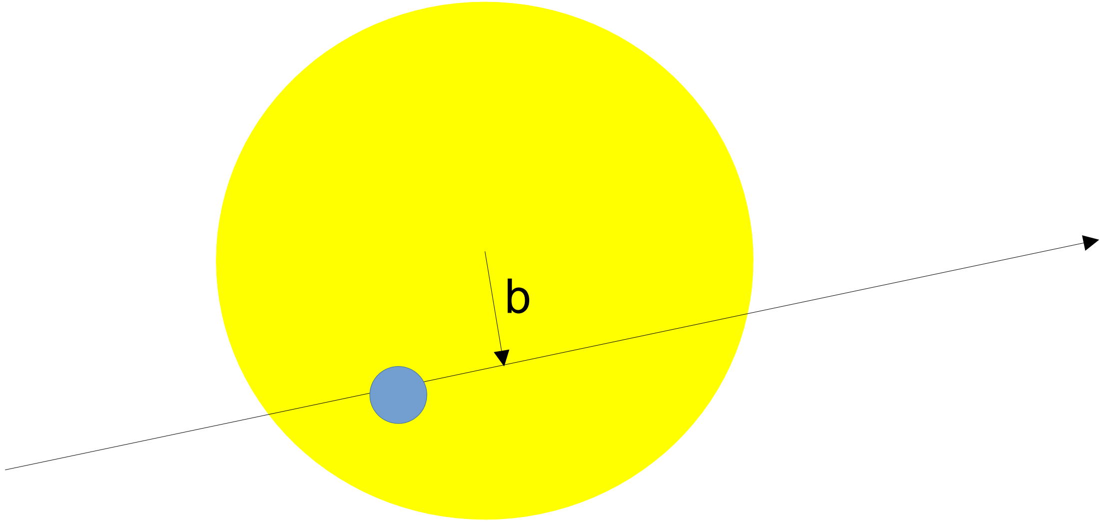




$$\xcancel{\frac{\Delta F}{F}=\left(\frac{R_{planet}}{R_{star}}\right)^2}$$

This equation is not true at all – because of limb darkening

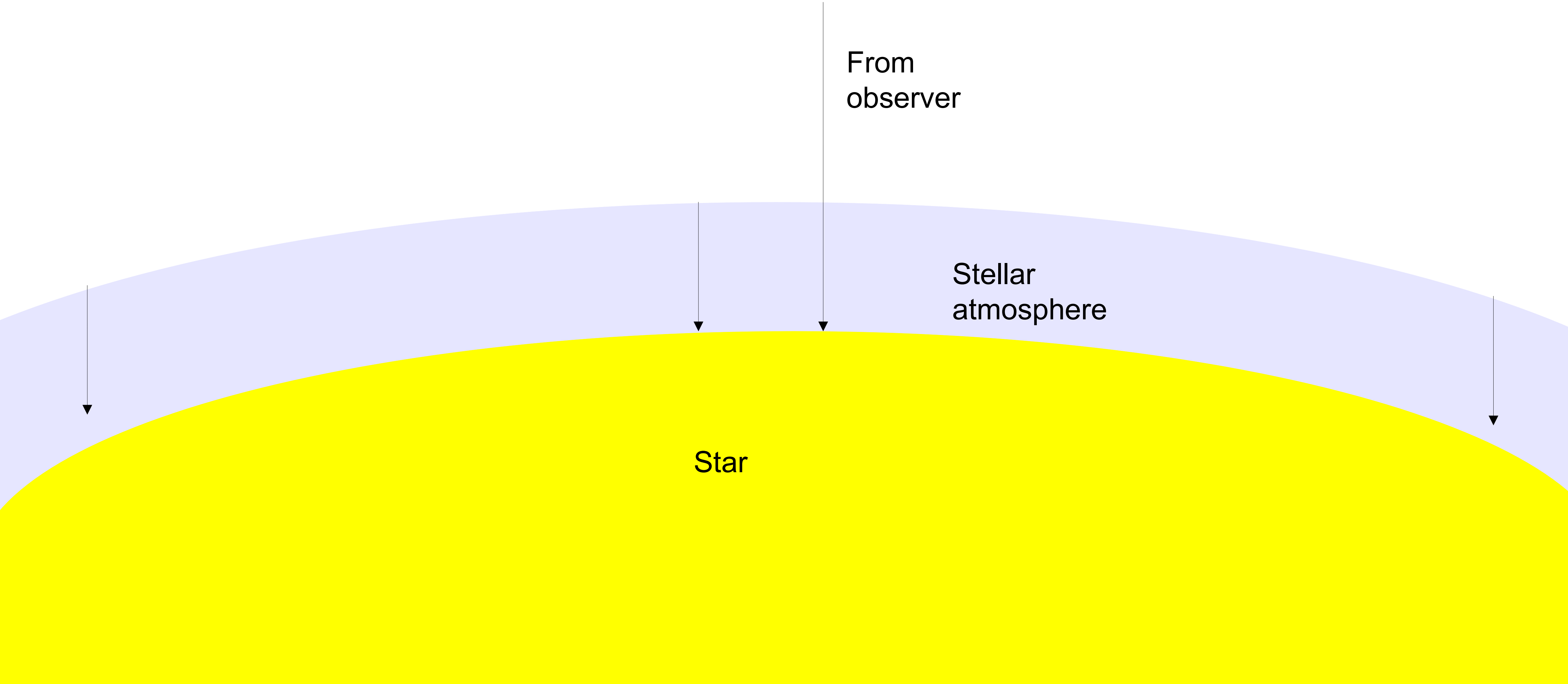




$$\frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}}\right)^2 L_D\left(b, u_i\left(T_{eff}, logg, Z, v_{micro}, geometry, P_{rot,star} \ldots\right)\right)$$

This equation is true – for spherical planet and star.
WP 114 must consider the dependence of b on e, ω, incl, a/R_{star}, …,
but this is not a topic of the present workshop.

Limb darkening/brightening in stars



History of limb darkening studies

Discovered in Sun by Luca Valerio (1612), communicated by Scheiner (1630).

Also mentioned by Galilei in a letter (1613).

First quantitative measurement: Bouguer (1729).

First theory: Schwarzschild (1906)

Already included to FIRST eclipsing binary star model in Russell (1912abc)

Luca Valerio (1553-1618)

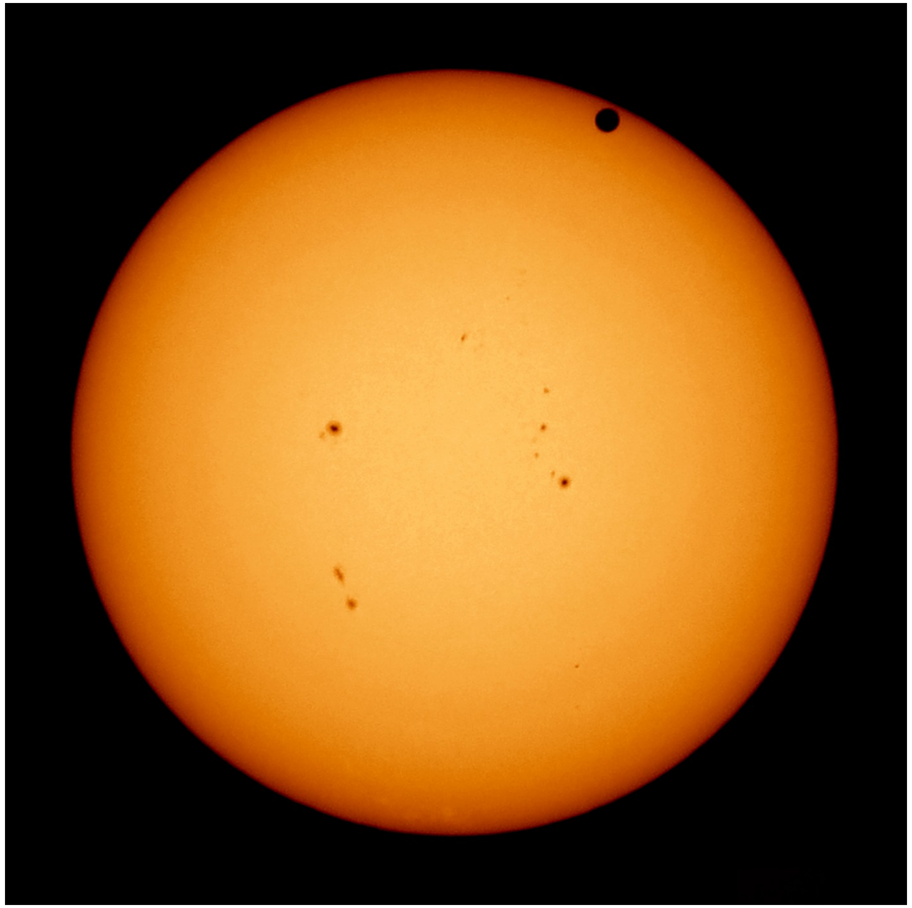
ApJ 36, 239 (1912)

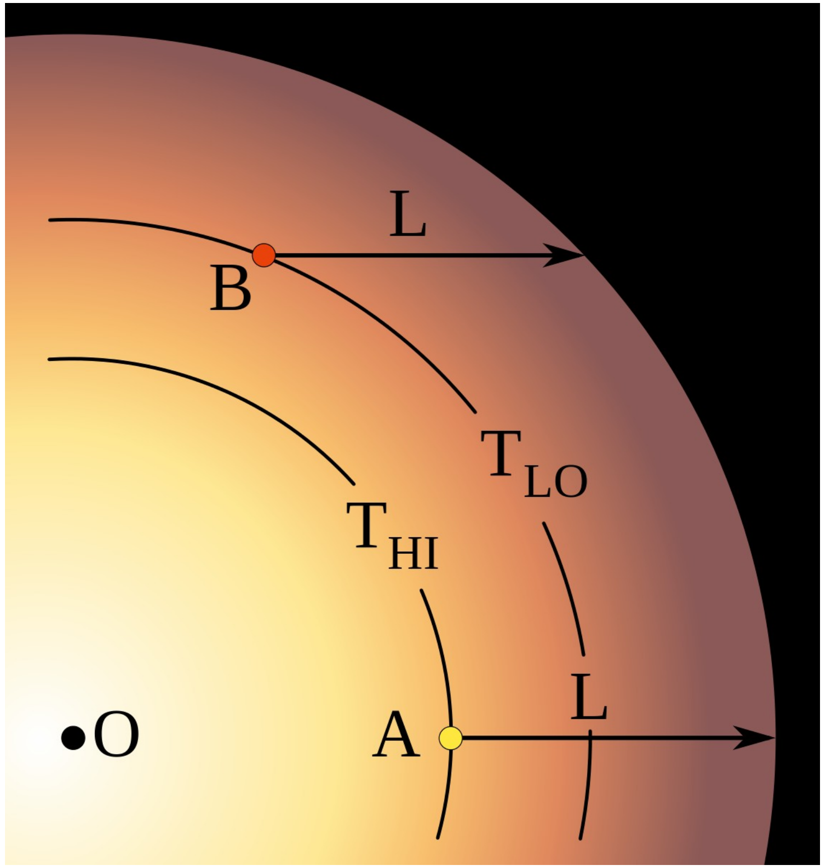
ON DARKENING AT THE LIMB IN ECLIPSING VARIABLES. I

By HENRY NORRIS RUSSELL and HARLOW SHAPLEY

1. *Solution for total eclipses of completely-darkened stars.*—In the problem of determining the orbital elements of an eclipsing binary from the light-curve, the nearly universal custom has been to assume that the stars are without appreciable absorbing atmospheres and that the stellar disks, therefore, are uniformly bright from the center to the limb. The present accuracy of photometric observations has been considered insufficient to justify introducing into the theory the complications which a contrary hypothesis would admit; moreover, the few reliable orbits so far computed on the assumption of uniform disks have represented the observations very satisfactorily. The extreme unlikelihood, however, of uniform disks has been generally recognized, the analogy of the sun being definite evidence in the case of stars of advanced spectral type, and attempts have been made by Rödiger[1] and Blažko[2] to determine the elements of β *Persei* and *U Cephei*, respectively, on the assumption of a differential darkening on the stellar disks equal

Limb darkening profiles and laws





SOHO and solar limb darkening

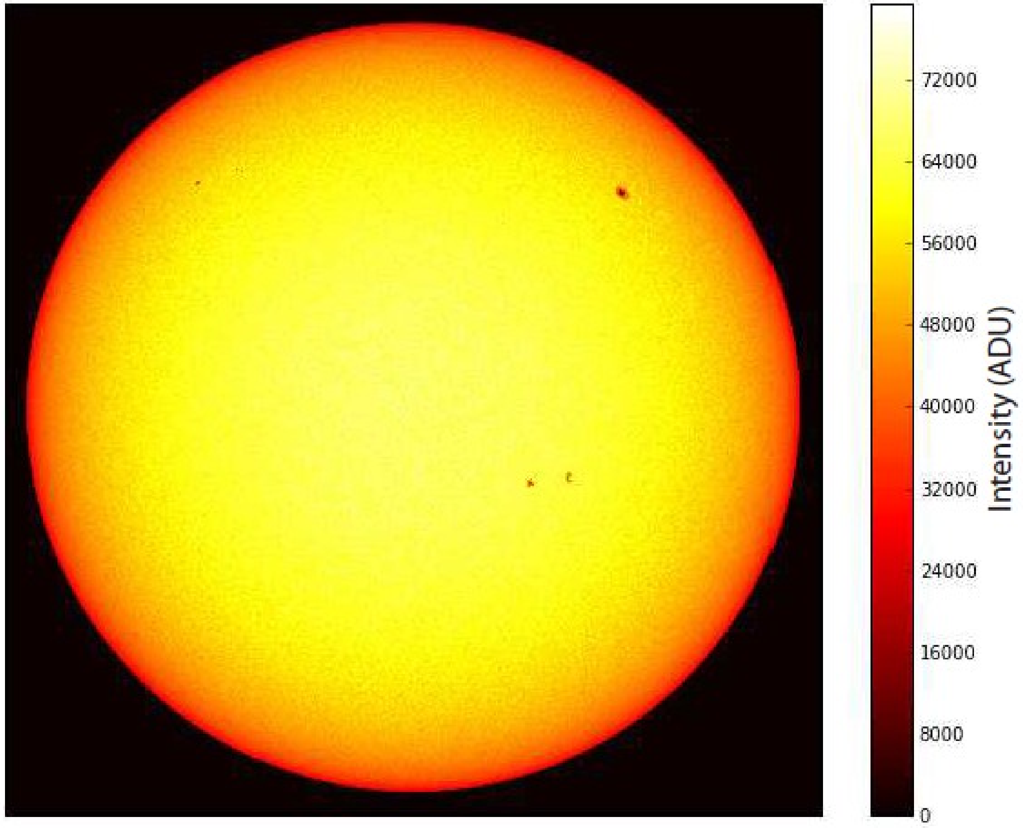


Fig. 1. Solar photosphere showing the limb darkening on January 3, 2011 (The solar image is taken by SDO/HMI).

Moon et al. (2017)

Do calculated spectra match with observations?

Figure from Pápics et al. (2011).

Black: observations

Gray: primary

Dashad gray: Secondary

RED: combined

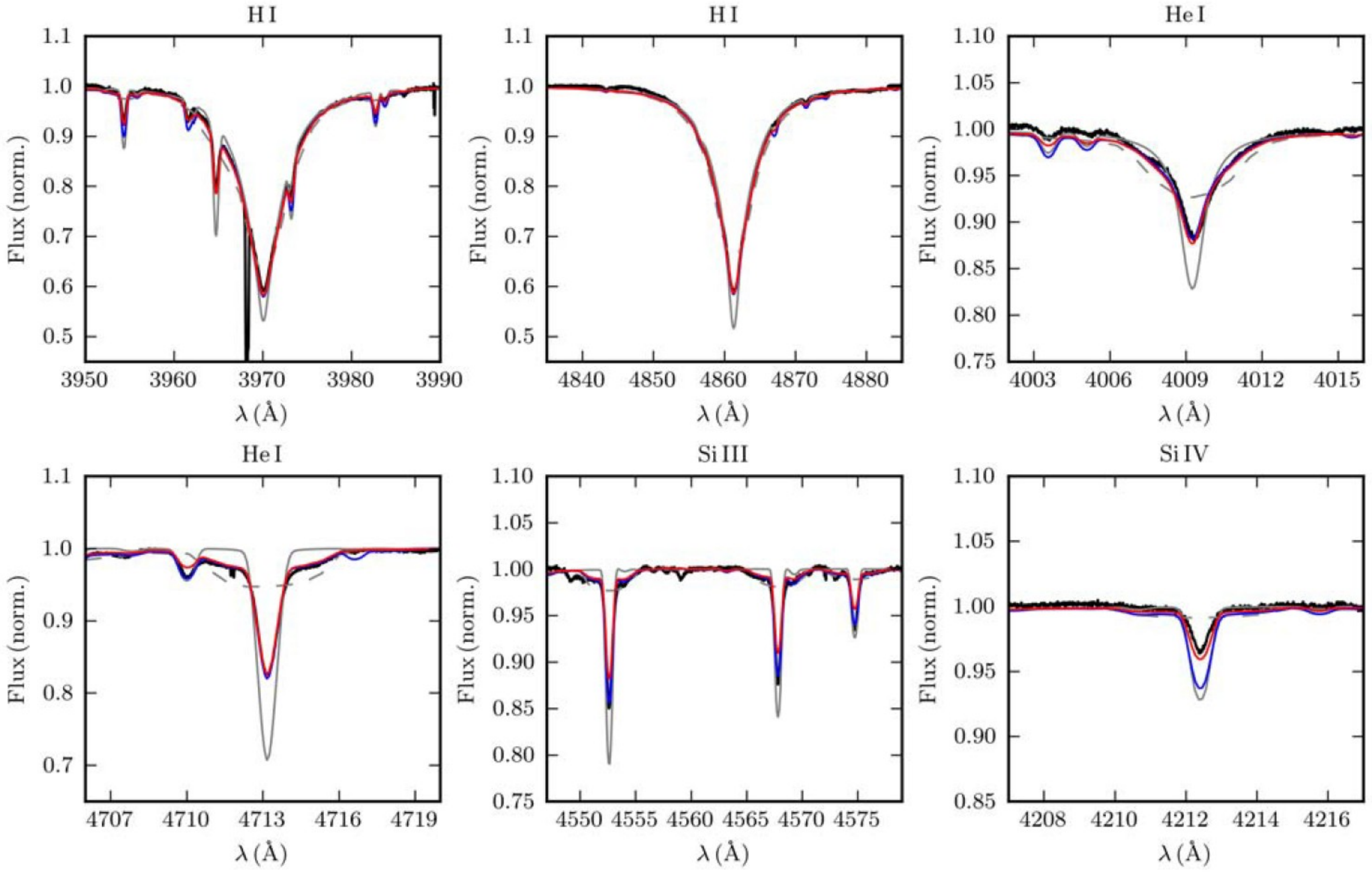
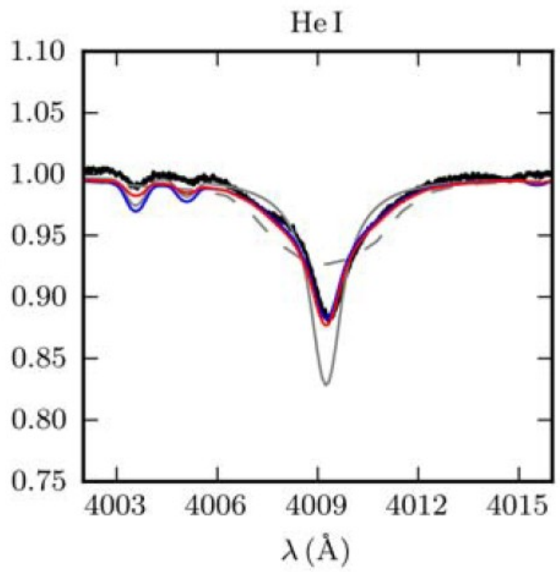


Fig. 2. Fits to the averaged HARPS spectrum (black). The parameters are listed in Table 2. The fit to the primary companion is shown in solid grey, the fit to the secondary in dashed grey, and the combined fit in red. While these synthetic spectra were calculated for $Z = 0.010$, we also plot the combined spectrum corresponding to $Z = 0.020$ with a solid blue line to show the effect of the change in metallicity.

Not a bad fit, but could be better (HI is OK, and look at Si III, IV):

What is the impact of spectrum-fits on determined
T_{eff}, logg, Z → limb darkening?

An expert of stellar spectra is needed for such studies and
extend this talk and our conclusions:
Does these deviations cause significant systematics in LD and
stellar parameters?
What improvement can be expected until 2026?



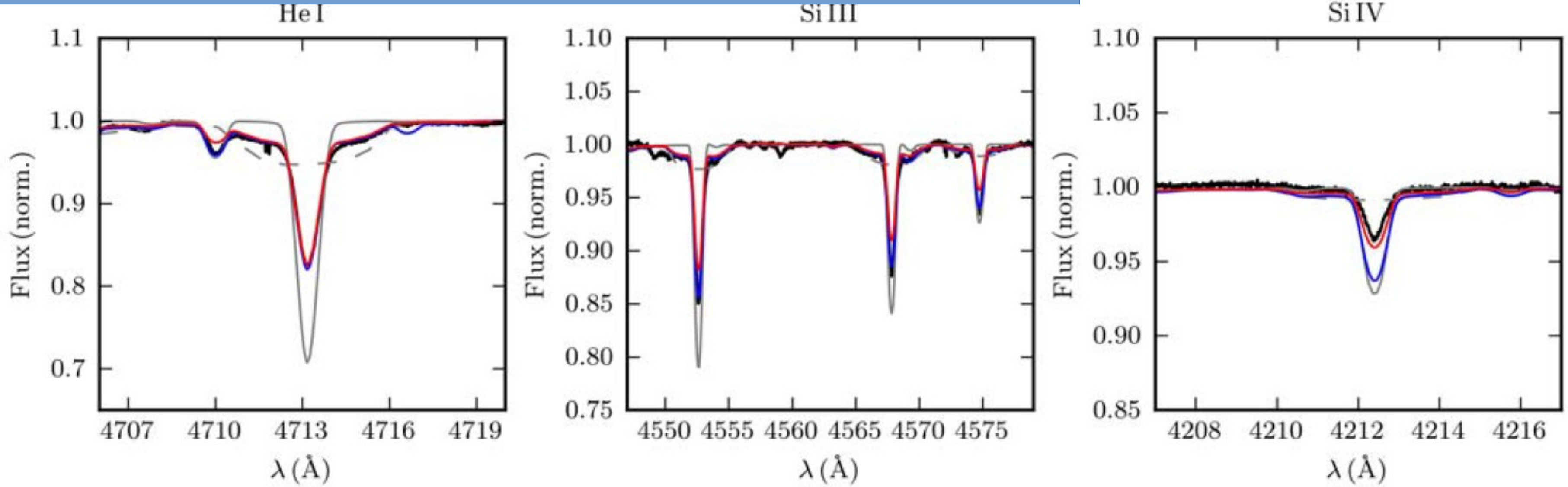
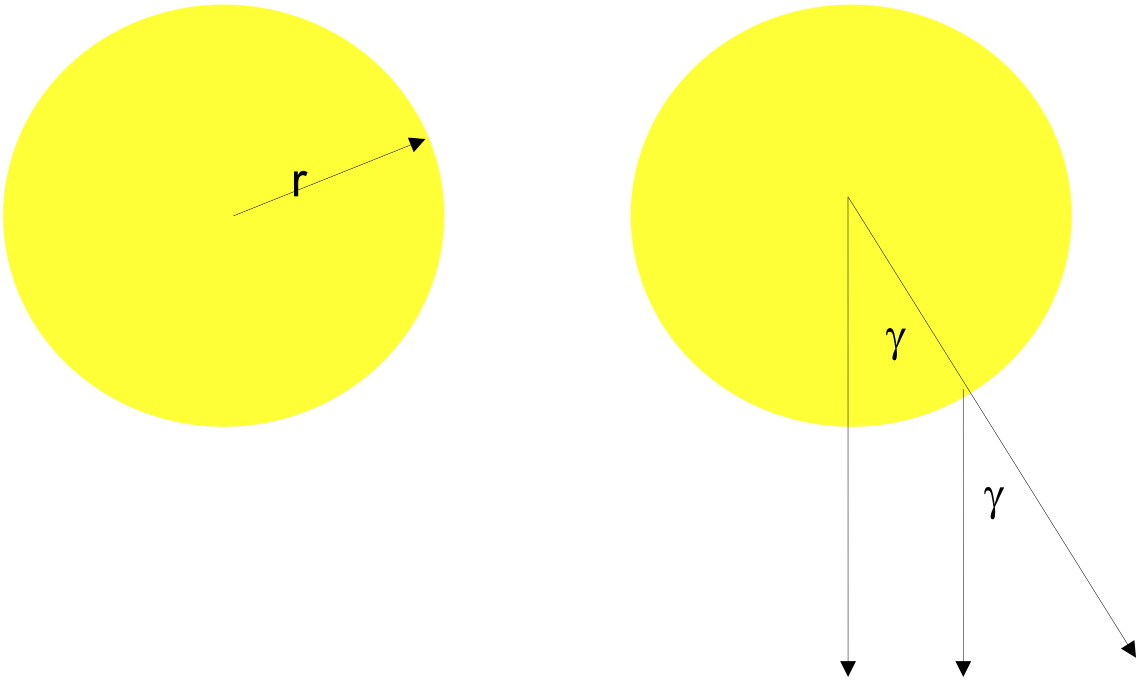


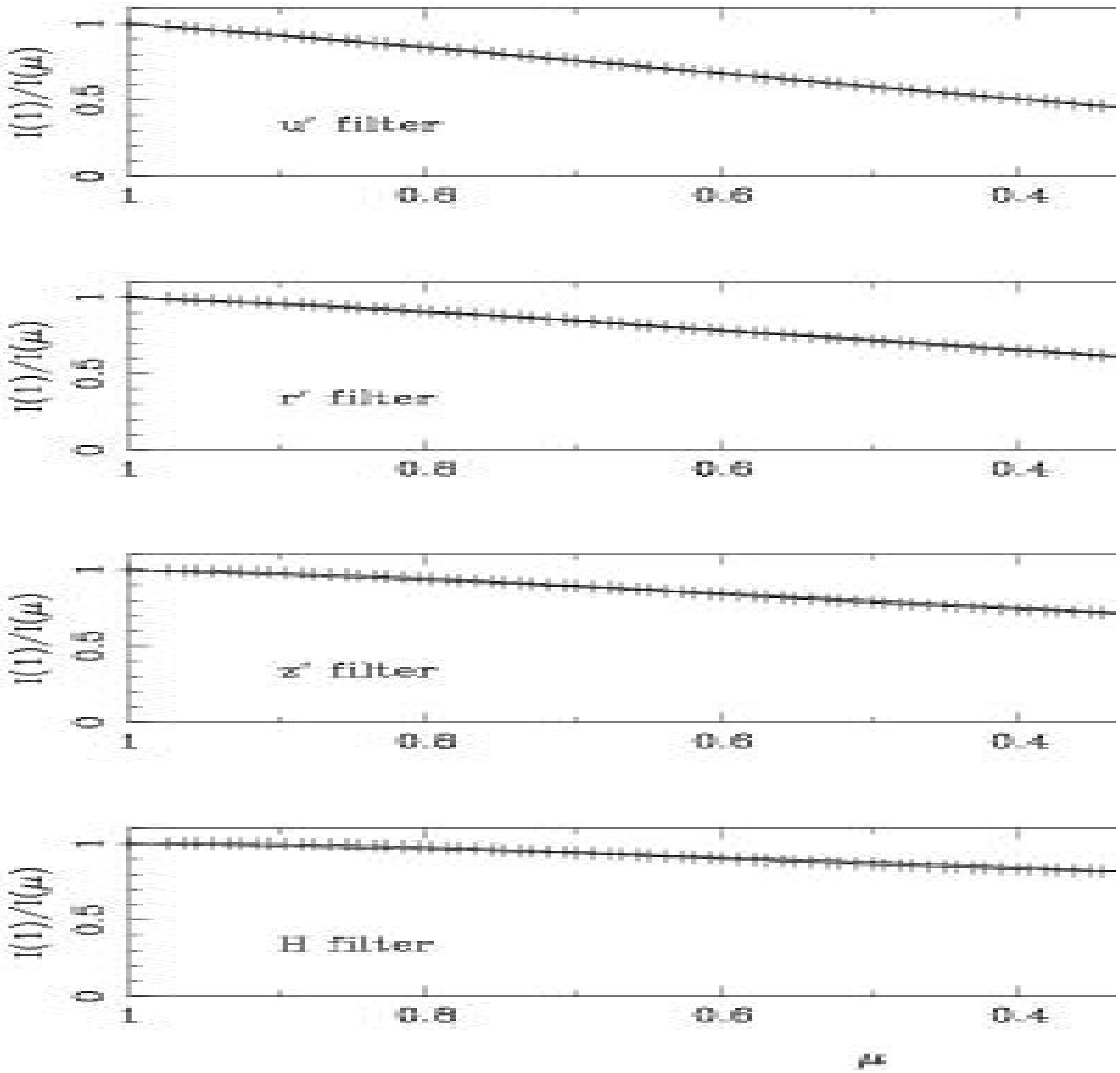
Fig. 2. Fits to the averaged HARPS spectrum (black). The parameters are listed in Table 2. The fit to the primary companion is shown in solid grey, the fit to the secondary in dashed grey, and the combined fit in red. While these synthetic spectra were calculated for $Z = 0.010$, we also plot the combined spectrum corresponding to $Z = 0.020$ with a solid blue line to show the effect of the change in metallicity.

DLR

From Claret, Hauschildt, Witte (2013, A&A 552, A16)

$$\mu = \cos\gamma = \sqrt{\left(1-(r/R)^2\right)}$$





DLR

Limb darkening “laws”

Linear: $$I = I(0)\left(1 - u(1-\mu)\right)$$

Quadratic: $$I = I(0)\left(1 - u_a(1-\mu) - u_b(1-\mu)^2\right)$$

Polynomial
(Csizmadia, unpublished) $$I = I(0)\left(1 - u_1(1-\mu) - u_2(1-\mu)^{40}\right)$$

Square-root: $$I = I(0)\left(1 - x(1-\mu) - y\left(1-\sqrt{\mu}\right)\right)$$

Logarithmic: $$I = I(0)\left(1 - a(1-\mu) - b\,\mu \ln \mu\right)$$

Exponential $$I = I(0)\left(1 - g(1-\mu) - \frac{h}{1-e^{\mu}}\right)$$ (Non-physical, see Espinoza and Jordan 2016)

Hestroffer & Magnan (1998)
Morello et al. (2017)
Maxted (2019) $$I = I(0)\left(1 - c\left(1-\mu^{\alpha}\right)\right)$$

What precision level do we need in LD?

$$\frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}}\right)^2 L_D$$

$$D = \frac{\Delta F}{F} = \left(\frac{R_{planet}}{R_{star}}\right)^2 \frac{1-u}{1-u/3}$$

$$\frac{\Delta\left(R_{planet}/R_{star}\right)}{R_{planet}/R_{star}} = \frac{1}{2}\frac{\Delta D}{D\sqrt{N_{transit}}} + f(u)\,\Delta u$$

~2
%

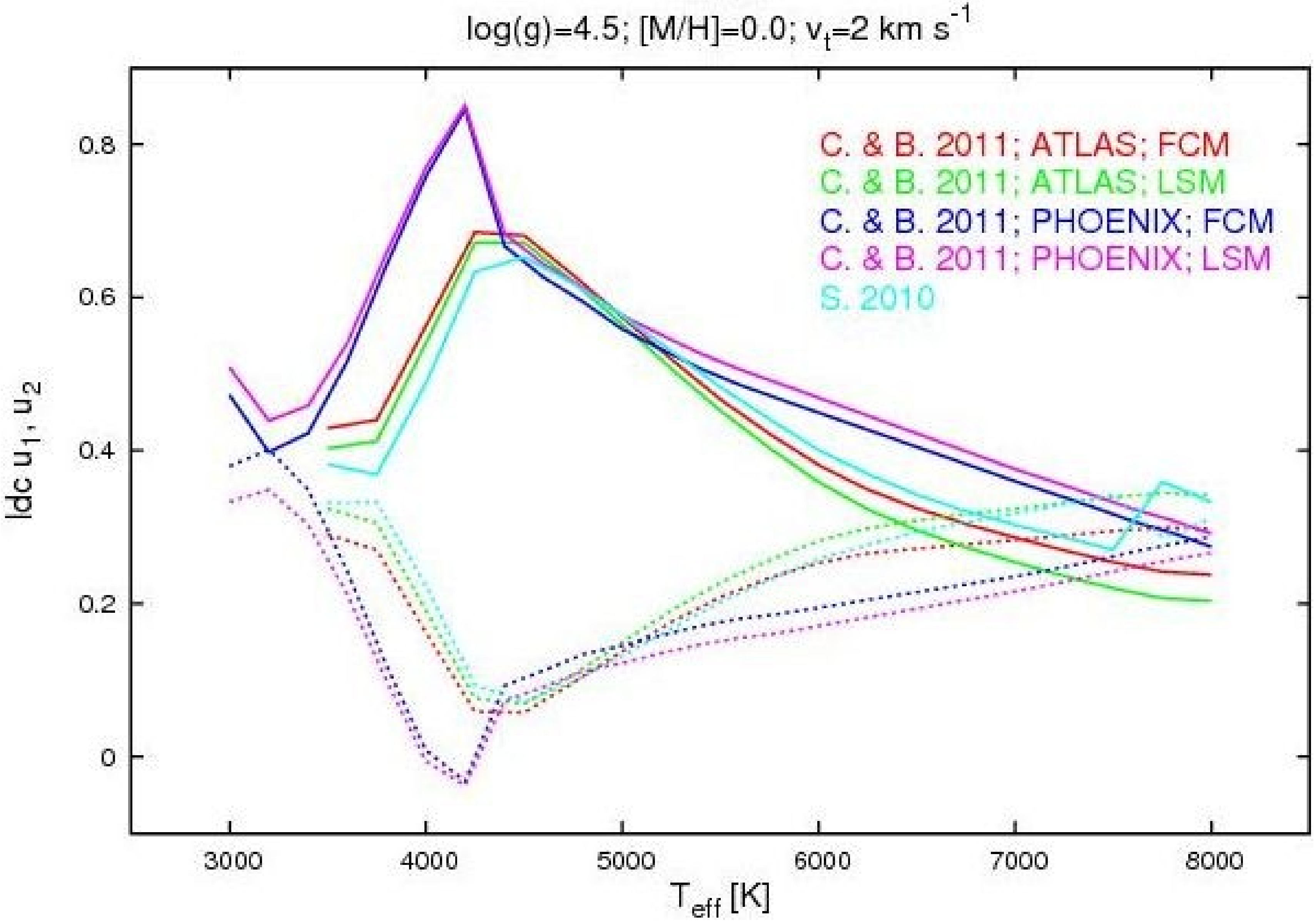
u ~0.6 for Sun

Simple estimation for Sun-Earth case

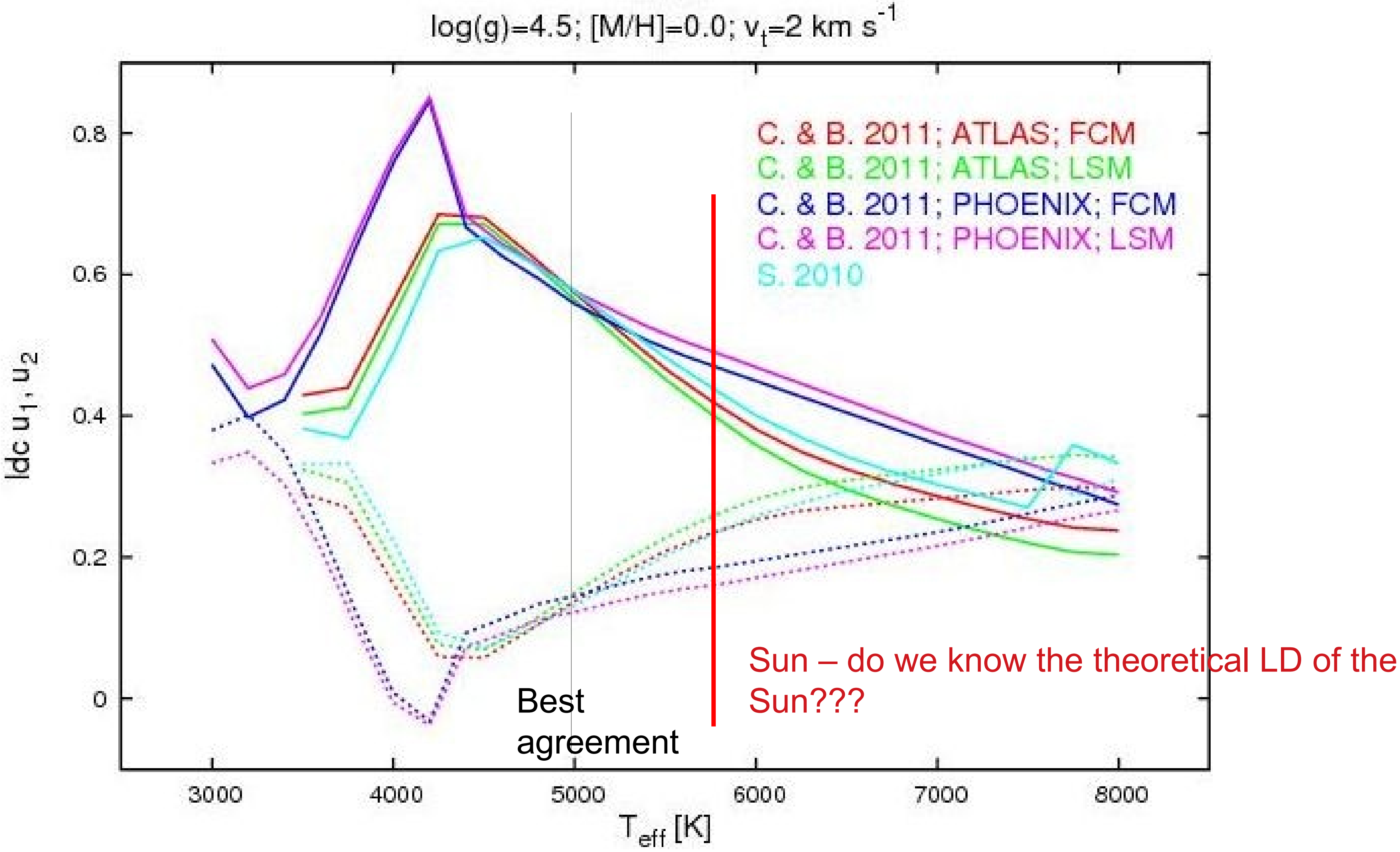
- 17% error in limb darkening coefficient cause 2% error in radius ratio (linear law used) → much, much higher accuracy is needed because of low number of observed transits

- But S/N is low if we observe only two transits at 1 year orbital period in a 2-years long pointing

- Transit depths is just 105 ppm, noise: 34ppm/hr$^{0.5}$

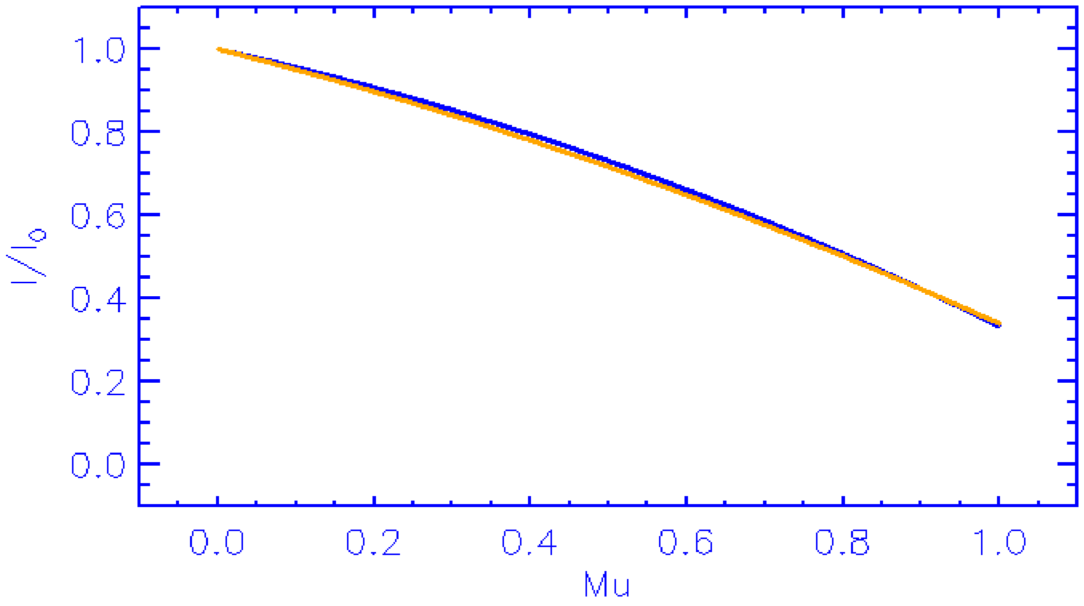


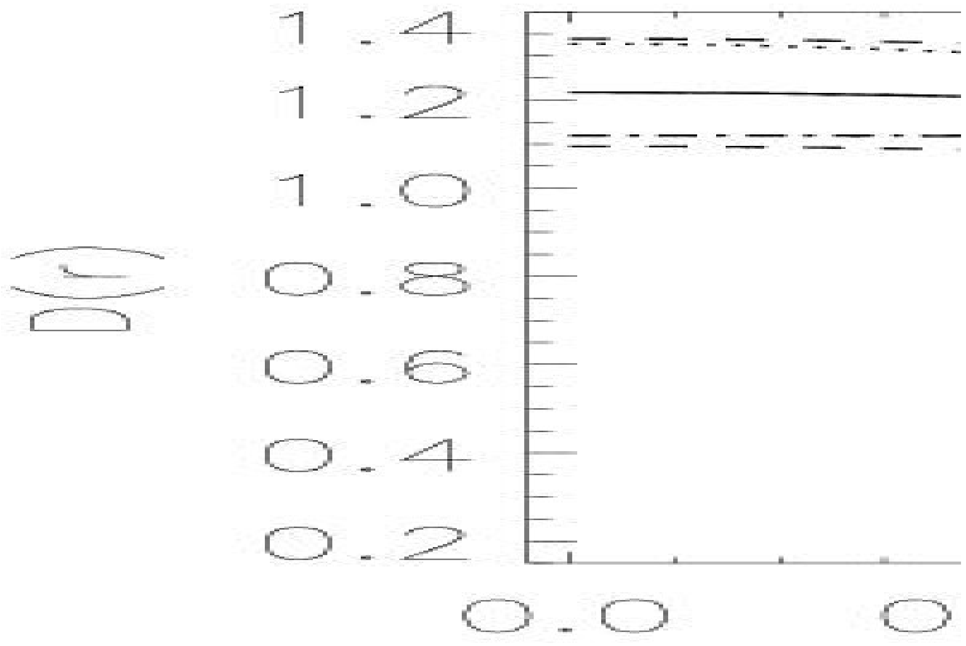
Comparison of Claret and Bloemen (2011) to Sing (2010), from Csizmadia et al. (2013)



Comparison of Claret and Bloemen (2011) to Sing (2010).

Intensity profile is more important than the values of LDCs due to degeneracies





From previous figure:

u1 = 0.4092, u2 = 0.2572

u2 = 0.4729, u2 = 0.1871

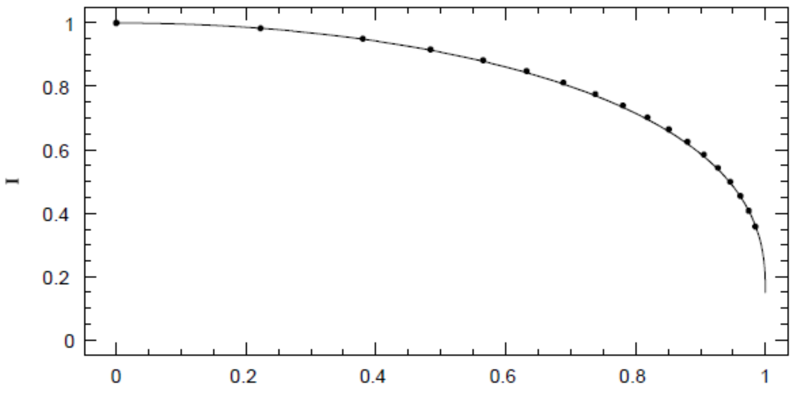
→ different LDCs can produce the same I(r)...

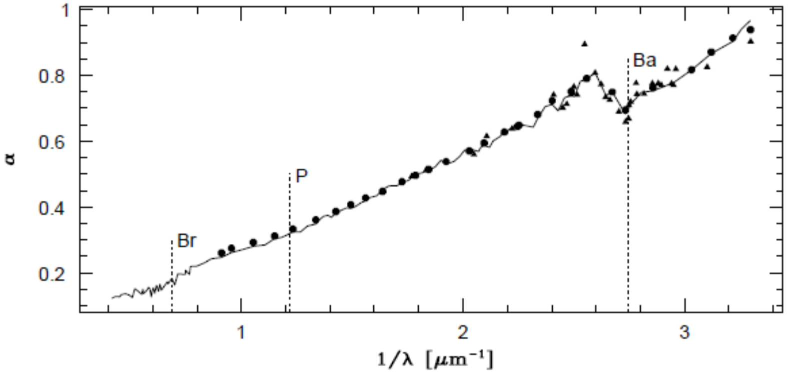
"The radial intensity profile D(r/R) as a function of the sky-projected distance r from the stellar centre (R is the stellar radius). The solid line shows the effect of limb darkening D(r) = LD(u1,u2,μ)/(1 − u1/3 − u2/6). The dashed lines show the tolerable ranges: between these lines the radial intensity distribution profile will produce a radius ratio k that is in the tolerance range of ±5%. The dotted line is an example of an acceptable radial intensity distribution profile with u1 = 0.82,u2 = −0.16, while the dot-dashed line is with u1 = 0.02, u2 = 0.6."
(Fig 7 from Csizmadia et al. 2013)

Stellar parameters and ld-predictions

- Usage of mean density from transits is essential (Torres et al. 2012) to avoid correlations between T_{eff}, Z, logg.
- Systematic shifts in T_{eff} between spectral analysis codes up to 80 K (Torres et al. 2012, others)
- T_{eff} values can be obtained sometimes ±70K →~±0.02 in LDCs
- Heavy spottedness leads to ~10% underestimation of radii (at $M<1M_{Sun}$) and overestimate of temperatures (Clausen et al. 2009)→ systematically shifted LDCs (Csizmadia et al. 2013)
- Does spotted areas have different LDCs? (E.g. Djurasevic 1992). If yes, LDCs can be negative/ over 1 causing limb brightening due to faculae at the limb or apperant Rayliegh-slopes (Csizmadia et al. 2013; Oshagh 2014). In this case LDC-tables and priors are useless and/or misleading.

The solar disc





Dots: measurements of the solar disc

Behaviour: exponent vs 1/lambda

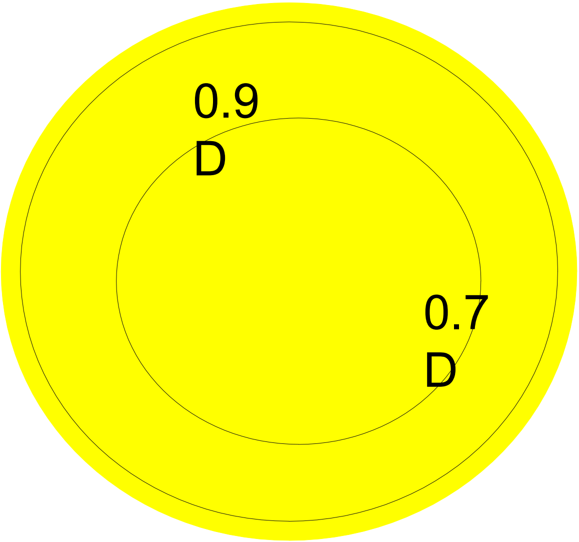
Fit: $I = I(0)\left(1 - 0.85\left(1 - \mu^{0.8}\right)\right)$ at some λ

Source: Hestroffer & Magnan (1998)

Asymmetry and periodic variations in solar limb darkening (or instrumental?)

- Periodic variation at 270 sec (Hill 1982, Koutchmy (1983), Yerle (1988)
- Yerle (1988): discussion of solar origin
- Neckel & Labs (1994):
  - No seasonal average variations from solar minimum to maximum (but Sun is relatively quiet star among all stars)
  - In certain intervals (sometimes <1 day), the actual LDCs can significantly differ from the average and they vary chaotically
  - 1-3% differences from the adopted average profile (10x bigger than that of needed for us) due to granulation, variable structure of supergranulation network, spots, flows, oscillations...
  - east-west asymmetry of LD with mostly unkown origin

Time-variable limb darkening in Sun… (Moon et al. 2017)



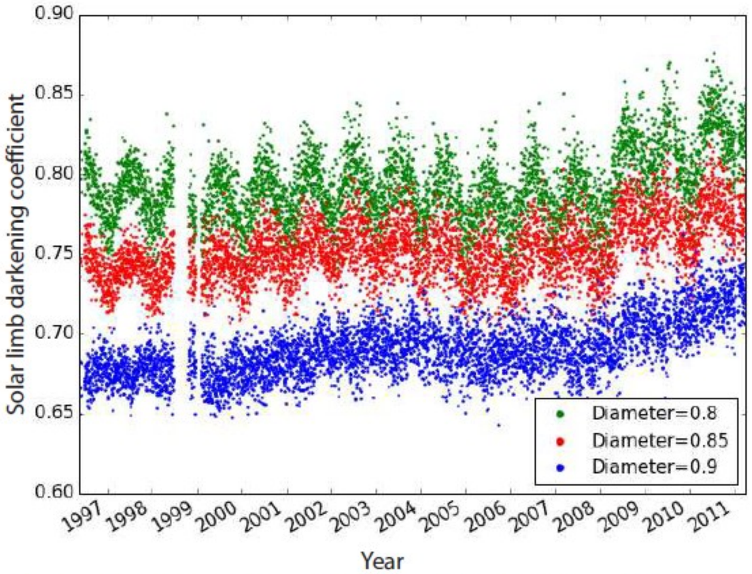


Fig. 6. Corrected daily coefficient of limb darkening estimated by photospheric intensity from solar images taken by SOHO.

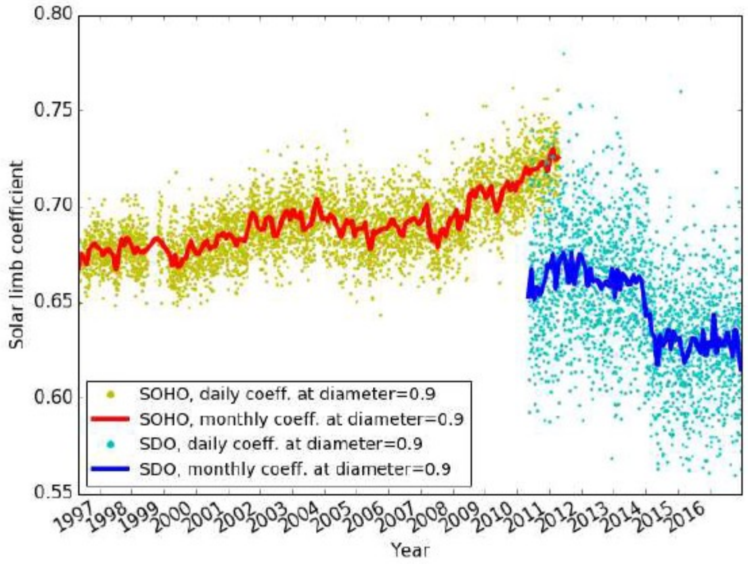


Fig. 8. Daily and monthly coefficients of limb darkening at the size of 0.9D.

A simple linear law was used in this work: $I = I_0(1 - u(1-\mu))$

Maximum 14% change in LDC, causing about 1.6-11% change in the derived radius ratio depending on impact parameter – ~same or higher than our accuracy/precision goal!

11-years spot cycle of Sun and LD

- Harder et al. (2011): 11-years spot modulation causes limb darkening variations (caveat: calibration problems)
- Criscouli and Foukal (2017): 11-years spot modulation causes NO limb darkening variations
- Hard to extrapolate to other stars (c.f. Djurasevic 1992) because spottedness often exhibits very different filling-factors in Sun and in other stars. I think issue is open.

Works useless in our field – or not???

- Debski (2015): “Observational verification of the limb darkening laws in contact binaries”
- In contact binaries there is only three free parameters because of the fixed geometry; mass ratio, inclination, fill-out
- But contact binaries rotates extreme fast, the atmospheric scale height as well as surface temperature is changing a lot from equator to pole (as large as ~1000K), so limb darkening is a highly variable with astro-latitude
- (But if we can model such complicated case, then extrapolating to simpler, slower rotating case can help? You can formulate your own doubts or positive view.)

Observational checks of the LD

- Sigismondi et al. (2015): using asteroid occultation to determine LD of Regulus
- Baines et al. (2014): 85 stars with interferometry
- Claret (2009): HD 209458 transit curve cannot be reproduced by any known (at that time) LD-calculation – does anyone know that new tables were applied to it?
- Claret (2008): eclipsing binaries show big deviations, scatter. Identified problem: input theoretical stellar atmosphere problems.

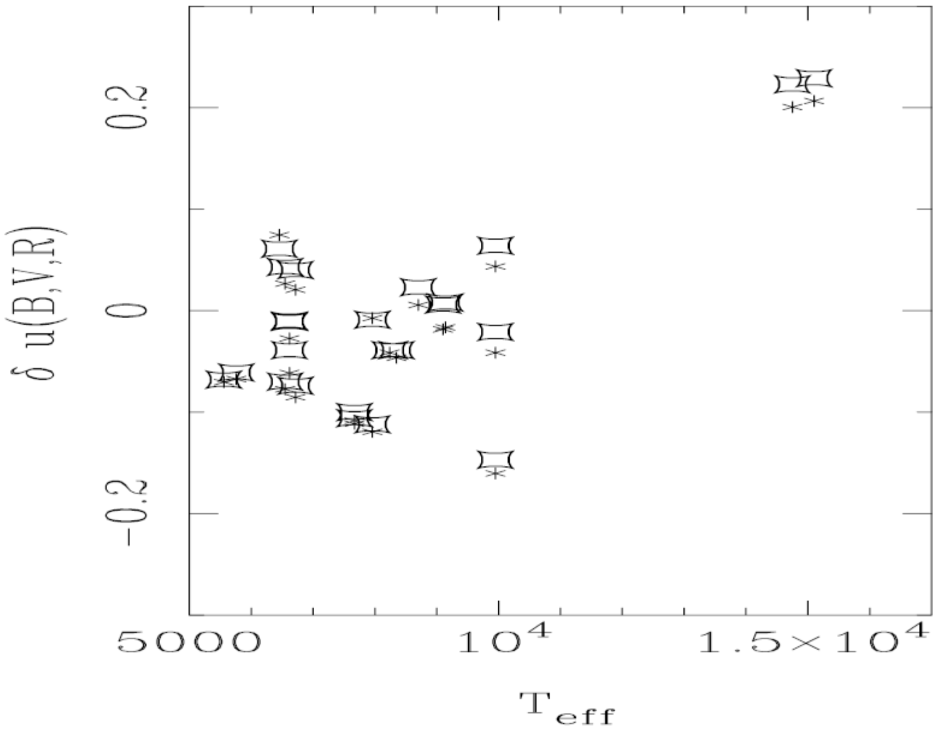
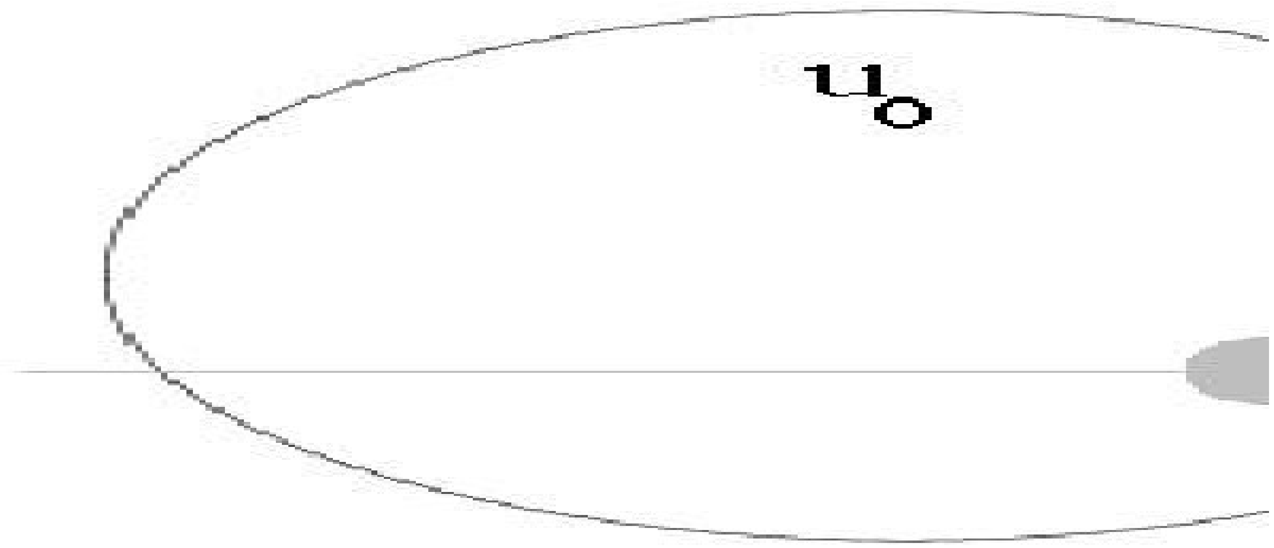


Fig. 4. The theoretical and observed linear LDCs for the nine eclipsing binaries. The open crosses and stars refer to the *B* and *V* bands, while full squares denote the *R* band. Only the theoretical error bars are shown.

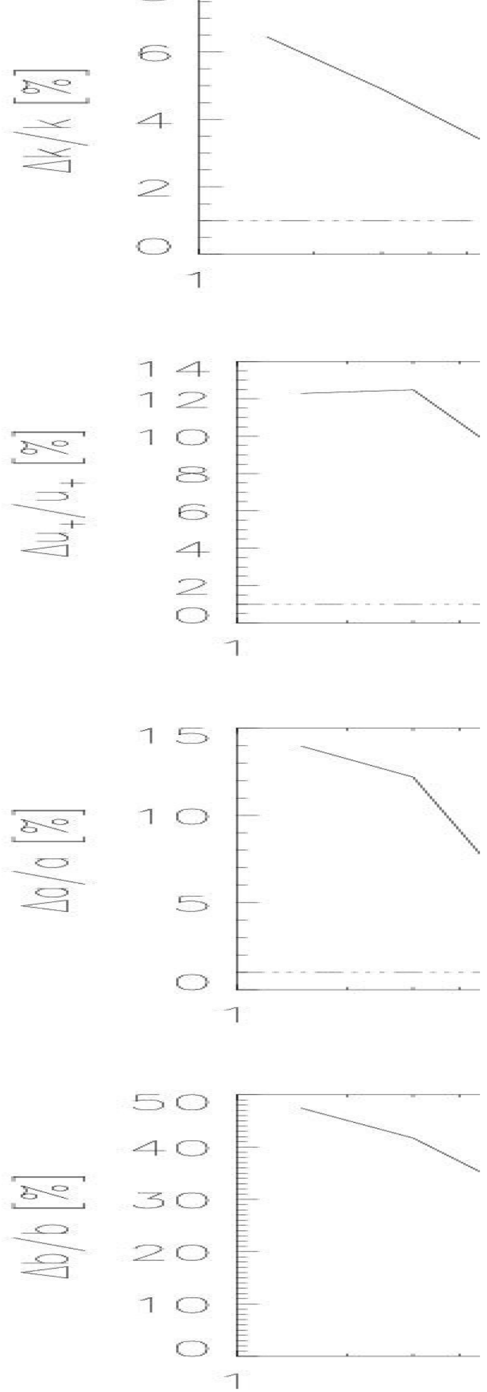
From Claret (2008).

Effect of stellar spots



Has no effects?
Do we use an area-weighted mean?

To fit or to fix? - “that is the question.”



Csizmadia 2013: to fit.
Müller 2013: fitted
Both studies identified problems with grazing transits, b>0.85.

Espinoza & Jordán (2016): fit, but something beyond linear/quadratic.
Recipes are given when it is better to fix.

Morris and Agol (2018): geometric use of spots, conclusion: to fit.

But what about S/N as a discriminator? When to use priors?

3D models of Magic et al. (2015)

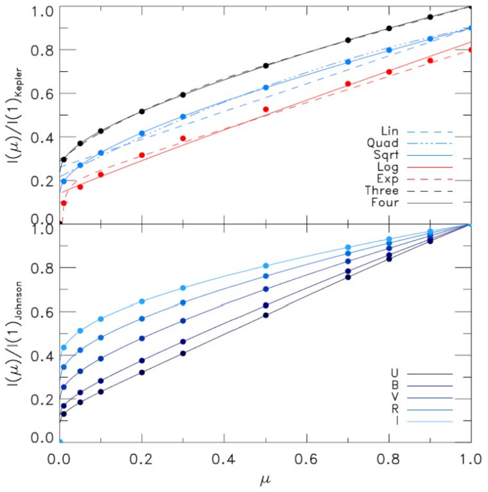


Fig. 3. Solar intensity distribution vs. μ-angle for different limb darkening laws in the Kepler filter (*top*) and different bands from the Johnson filters (*bottom panel*). The 3D results are indicated with filled circles. In the *top panel* the blue and red lines are shifted by 0.1 and 0.2 respectively, while the black lines are unshifted.

Magic et al. (2015):

Only the four-parameter law performs well
- systematic differences are large enough to prefer 3D models

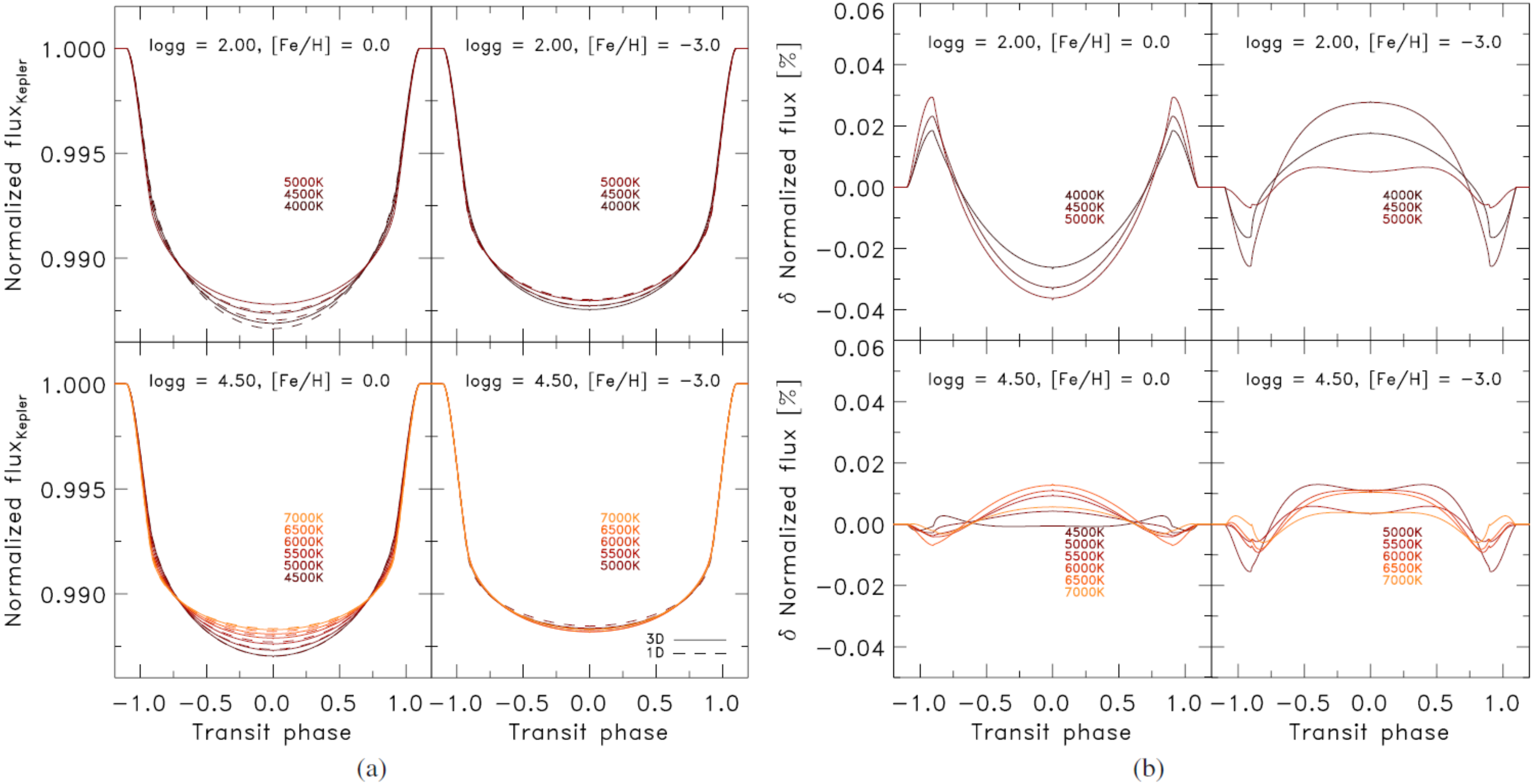


Fig. 8. *Left figure*: transit light curve vs. transit phase with $p = 0.1$ in the Kepler filter for different stellar parameters. The predictions from the 1D ATLAS models are also included (dashed lines). *Right figure*: relative deviations in the transit light curve with $p = 0.1$ between 3D atmosphere models and 1D ATLAS models given in %. The difference is $\delta = 1D/3D - 1$.

Plane parallel vs spherically symmetric models of Neilson and Lester (2013)

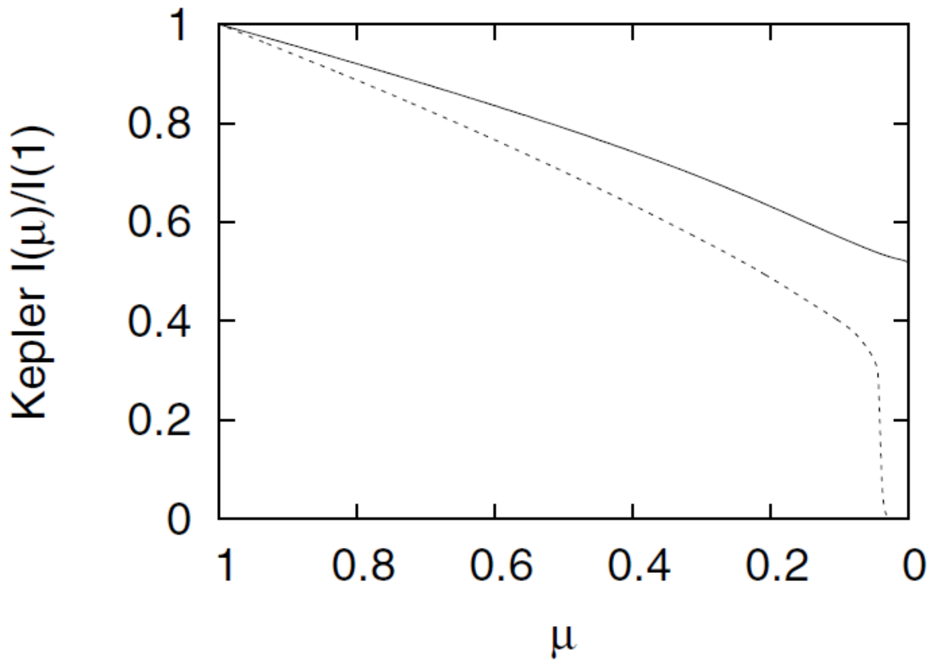


Fig. 1. Predicted *Kepler*-band intensity profiles for plane-parallel (solid line) and spherically symmetric (dotted line) model stellar atmosphere with $T_{\rm eff}$ = 5800 K, $\log g$ = 4.5 and M = 1.1 $M_\odot$.

DLR

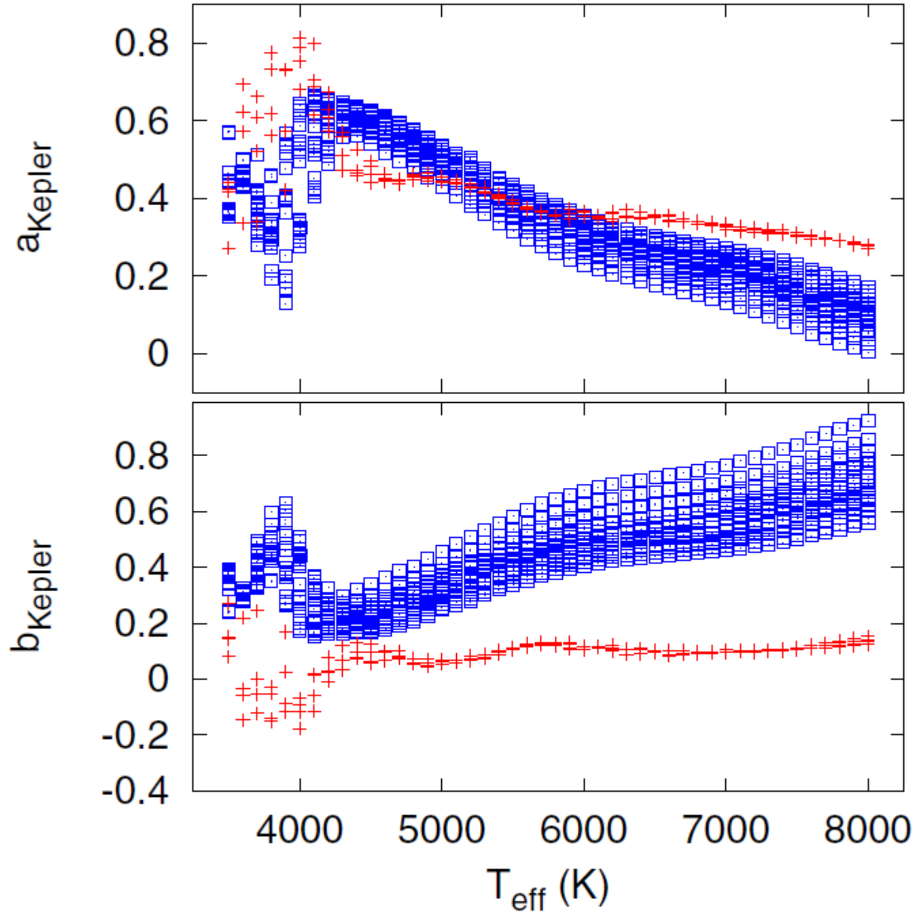
Plane parallel vs spherically symmetric models of Neilson and Lester (2013)

Quadratic coefficients:

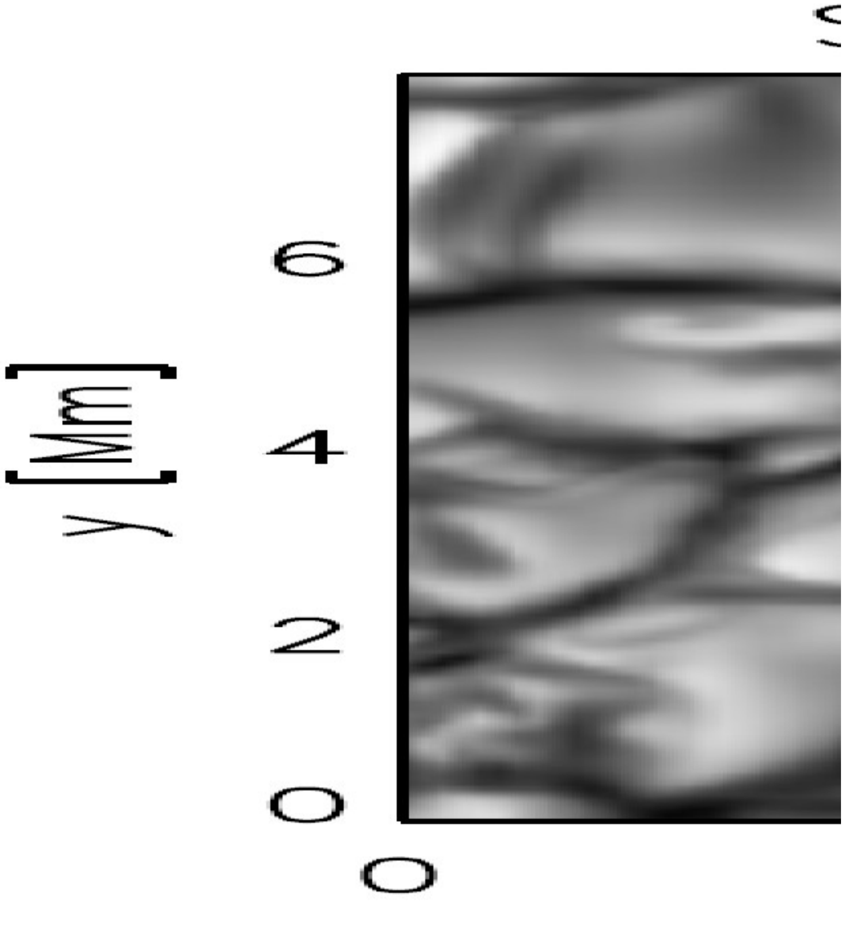
Red crosses: plane parallel

Blue squares: spherically symmetric

Their paper gives the difference for other LD-laws as well.



Granulation (Chiavassa et al. 2017)



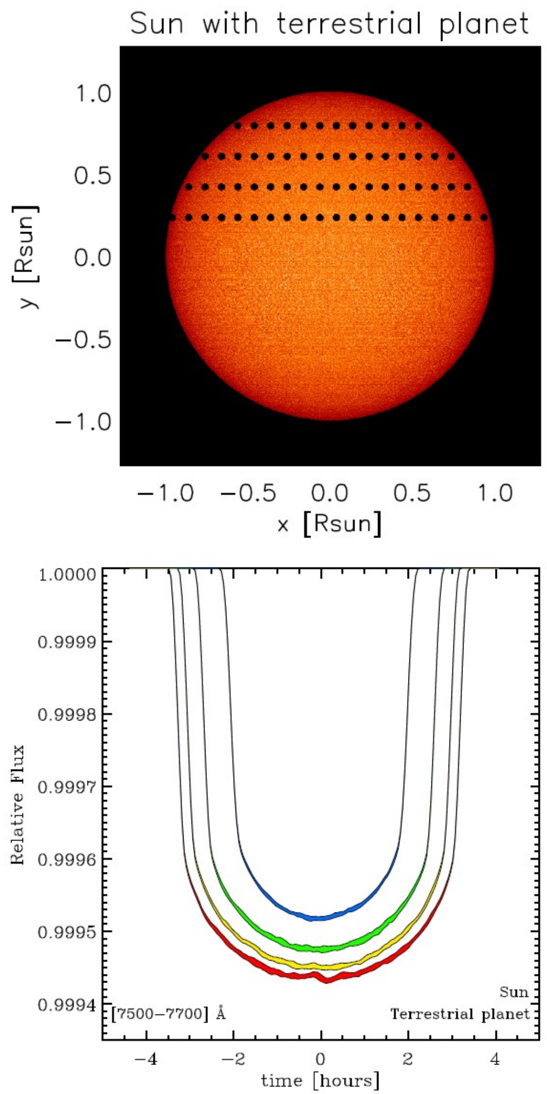


Fig. 11. *Top panel*: different transit trajectories of the prototype planet *Kepler*-11 on the Sun at the representative wavelength band of [7620–7640] Å and for four orbital inclination angles *inc* = [90.85, 90.65, 90.45, and 90.25]° (*from top to bottom* transit). An inclination angle of 90° corresponds to a planet crossing at the stellar center (Fig. 8). *Central panel*: transit light curves with colored shade denoting highest and lowest values of 42 different synthetic images to account for granulation changes during the transit. Blue corresponds to *inc* = 90.85°, green to 90.65°, yellow to 90.45°, and red to 90.25°.

Granulation (Chiavassa et al. 2017)

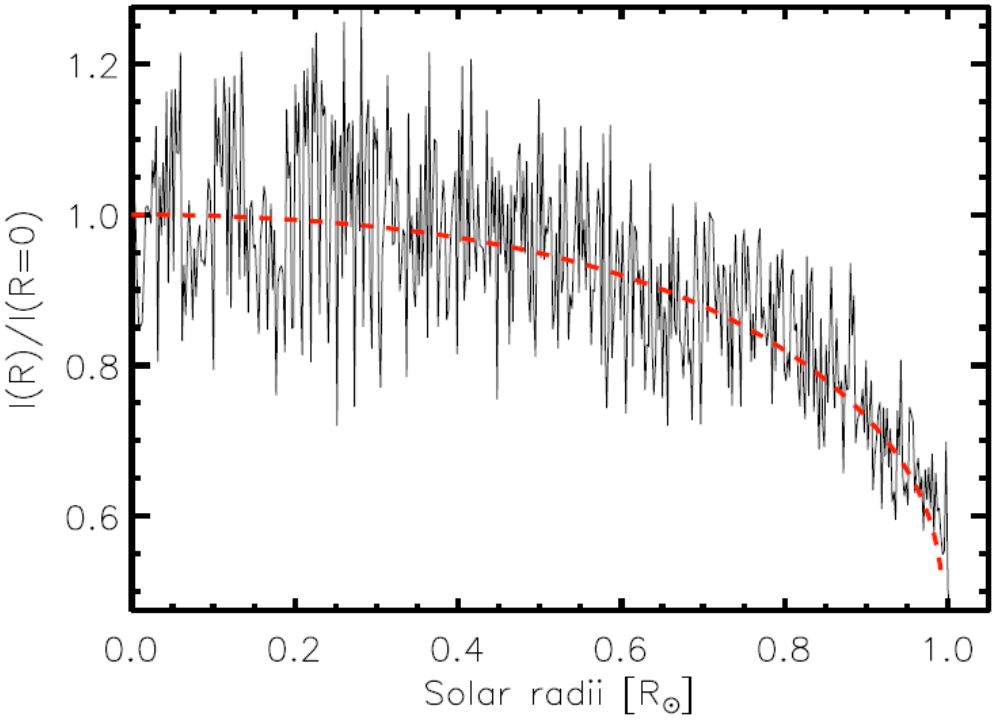
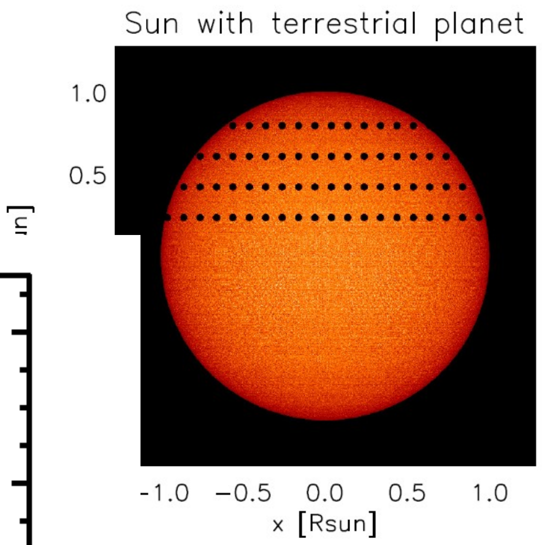


Fig. 12. Example of limb-darkening fit (red line) with the Claret law (see text) to the averaged intensity profiles of Fig. 4.



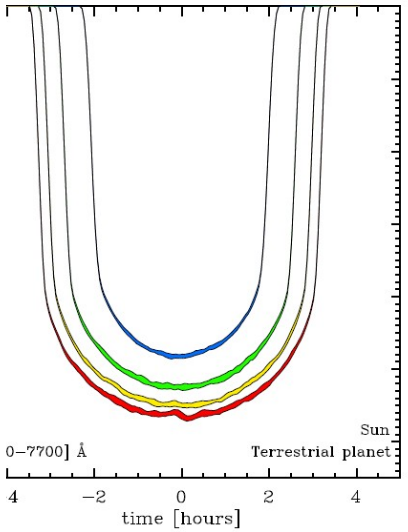
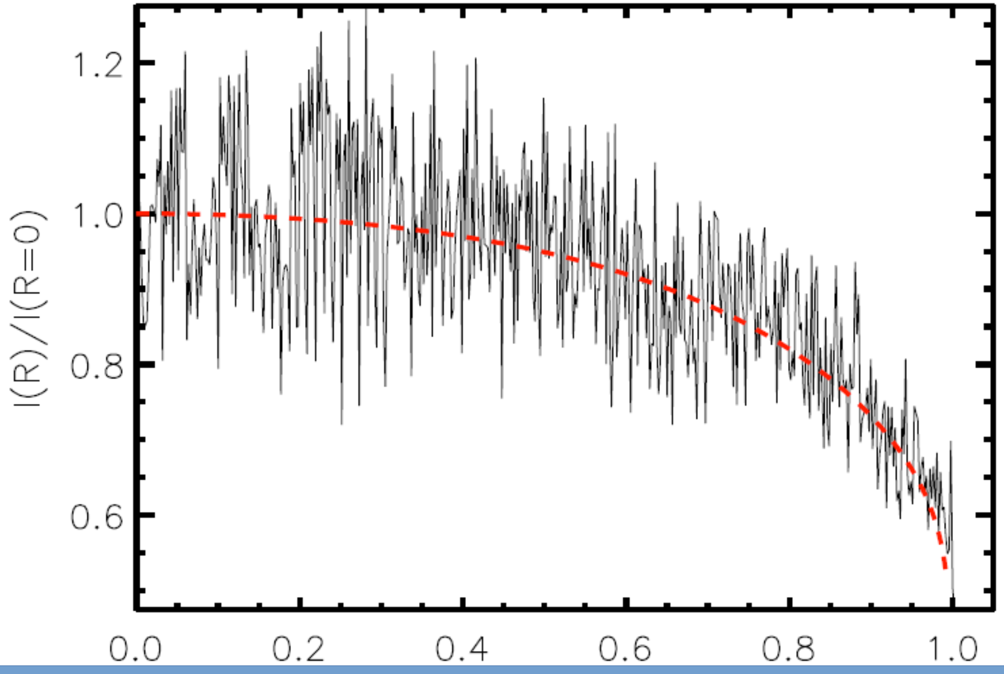
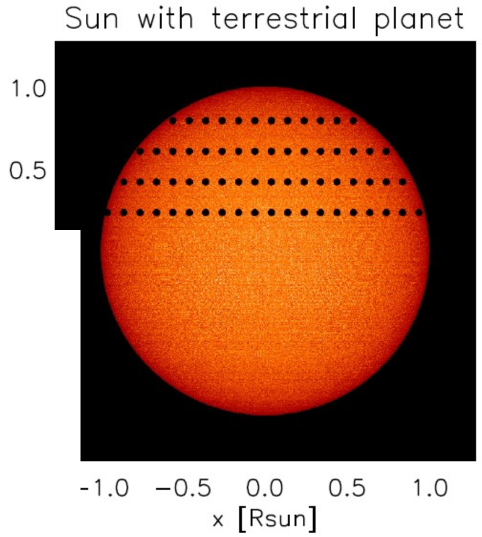
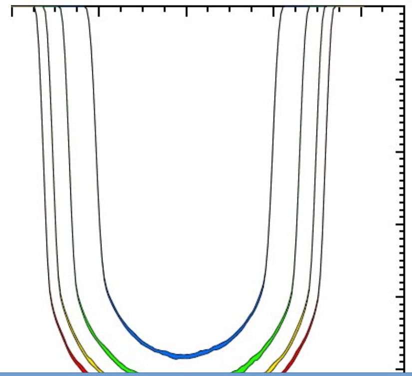


Fig. 11. *Top panel*: different transit trajectories of the prototype planet *Kepler*-11 on the Sun at the representative wavelength band of [7620–7640] Å and for four orbital inclination angles *inc* = [90.85, 90.65, 90.45, and 90.25]° (*from top to bottom* transit). An inclination angle of 90° corresponds to a planet crossing at the stellar center (Fig. 8). *Central panel*: transit light curves with colored shade denoting highest and lowest values of 42 different synthetic images to account for granulation changes during the transit. Blue corresponds to *inc* = 90.85°, green to 90.65°, yellow to 90.45°, and red to 90.25°.

Granulation (Chiavassa et al. 2017)







This kind of noise can be averaged out by adding many transit together.

But do not forget: we'll observe only 2-4 transits for many of the long-period exoplanets with PLATO!

And PLATO is a discovery as well as a characterization mission.

Limb darkening-free?

- Heller (2018): using midtransit and average transit depth to determine LD with the (very) small planet approximation
  - Advantage: can give priors to the numerical work
  - Criticism: works only for very tiny planets, required S/N ratio is not explored (probably it kills the method), may depend on the chosen ld-law to invert the mean/mid transit depths
  - I am not sure wether detailed numerical work does the same or not, but it helps to check the consistency of the results
- Morris et al. (2018): utilization of geometry of spots; findings support to fit LD in transit light curve analysis.
- See also Csizmadia et al. In prep. - also works only for high S/N

Summary and recommendations

(1) Theoretical difficulties in understanding of limb darkening → convection in stellar atmospheres,input physics of stellar spectra, 3D models, spherical symmetric models, processes on time-scales of transits and longer (granulation, variability etc)

(2) More precise stellar temperatures, logg, Z values are needed

(3) Study further the behaviour of LD of Sun

(4) More observational check on wide eclipsing binaries and on transiting exoplanet systems, time-variability can be interesting

(6) Fit or fix: depends on S/N-ratio. Understand what we fix in LD...

(7) Using intensity profiles instead of laws? → numerical models replace analytic ones (speed)

(8) Closer relationship between relevant groups: WP 114 (Transit Tools), WP 122 (1D, 3D, fundamental stellar parameters, Limb Darkening subpackages), WP 146 (Hatzes, Spectroscopy and 146 100: activity indicators, and 146 200: spectral classification)

Back-up slides

Recent limb darkening tables not listed in Csizmadia (2018)

- Reeve & Howarth (2016, MNRAS)

Can we reach this precision?

- List of factors to be considered (not a full list):
  - Baseline-variations (stellar activity, variability, instrumental)
  - Stellar parameters
  - Limb-darkening:
  - Stellar spectra precision
  - Plane parallel vs spherical geometry
  - 1d vs 3D stellar atmosphere models
  - Granulation at different scales
  - Dynamical vs static atmosphere models (pulsation)
  - Stellar spots
  - LD-laws vs direct profile fitting